Day 2 Applications

Note Title 9/13/2008

2.1 Entanglement in flat spacetime for inertial and uniformly accelerated observers

2.2 Entanglement in curved spacetime (Toy model for exanding universe)

Consider a scalar fiel $\phi(t,x)$ defined in every point of a Minkowski spacetime (flat spacetime) 1+1 dimensions
line element is given by

$$ds^2 = dt^2 - dx^2$$

Killing vector fields are found by solving the equation

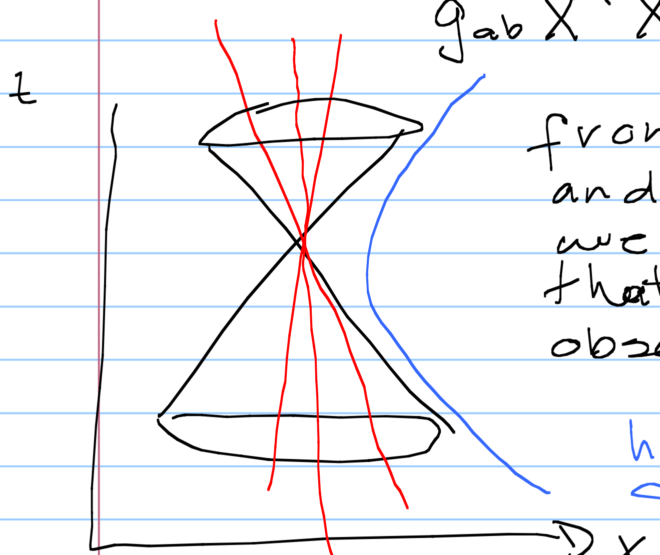
$$\mathcal{L}_X g_{\alpha\beta} = 0$$

$$X^c \frac{\partial}{\partial x^c} g_{ab} + \frac{\partial X^c}{\partial x^a} g_{cb} + \frac{\partial X^c}{\partial x^b} g_{ac} = 0$$

3 independent Killing vector fields

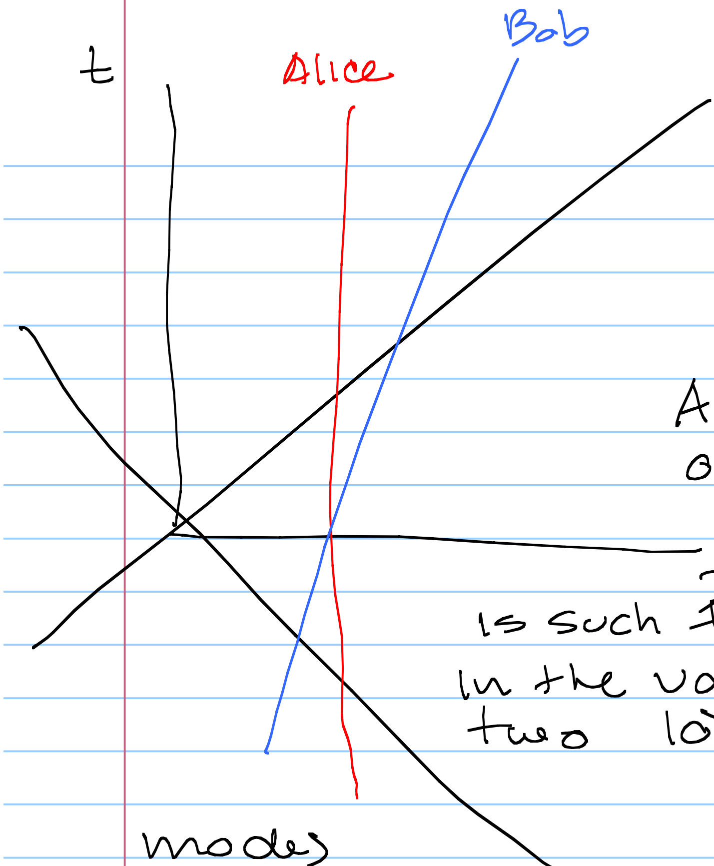
- time translation
- space translations
- observers moving on hyperbolas

$$g_{ab} X^a X^b < 0$$



from time translations and space translations we obtain timelike trayectories that correspond to inertial observers

hyperbolic trajectories correspond to observers moving with uniform acceleration



(t, x) Minkowski coordinates

Inertial frame

Alice and Bob are two observers

The state of the field is such that all the modes are in the vacuum state except from two $|0\rangle = |0\rangle_{k_1} \otimes |0\rangle_{k_2} \otimes |0\rangle_{k_3} \dots$

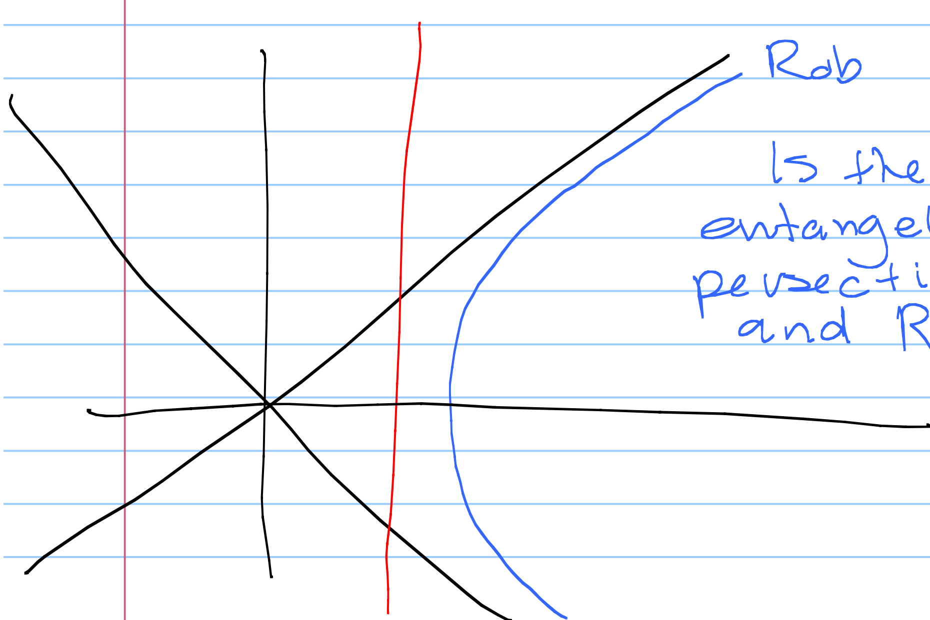
modes k and k'

are in the state $|\psi\rangle_{kk'} = \frac{1}{\sqrt{2}}\left(|0\rangle_{k_1}|0\rangle_{k_2} + |1\rangle_{k_1}|1\rangle_{k}\right)$

$|\psi\rangle_T = |\psi\rangle_{kk'} \otimes |0\rangle_\cdot |0\rangle_\cdot \dots$

Alice has a detector sensitive to k
Bob " " " " k'

Their measurements are prefectly correlated



Is the state maximally entangeldu from the persective of Alice and Rob?

massless

$\phi(t,x)$ $\quad ds^2 = dx^2 - dt^2$

We saw that the Klein-Gordon equation is

$$\left(\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2}\right)\phi = 0$$

solutions: plane waves

$$u_k = \frac{1}{\sqrt{2\pi\omega}} e^{ikx - i\omega t} \qquad \omega = |k| \qquad -\infty < k < \infty$$

$$u_k^*$$

$i\partial_t u_k = \omega_k u_k$ positive frequency modes

$i\partial_t u_k^* = -\omega_k u_k$ negative " "

$$\frac{\partial}{\partial t} \equiv \partial_t$$

$$\phi = \sum_k u_k^* a_k + u_k a_k^\dagger$$

$$a_k|0\rangle = 0$$

Minkowski vacuum

$$|1\rangle_k = a_k^\dagger|0\rangle$$
$$|1\rangle_{k'} = a_{k'}^\dagger|0\rangle$$

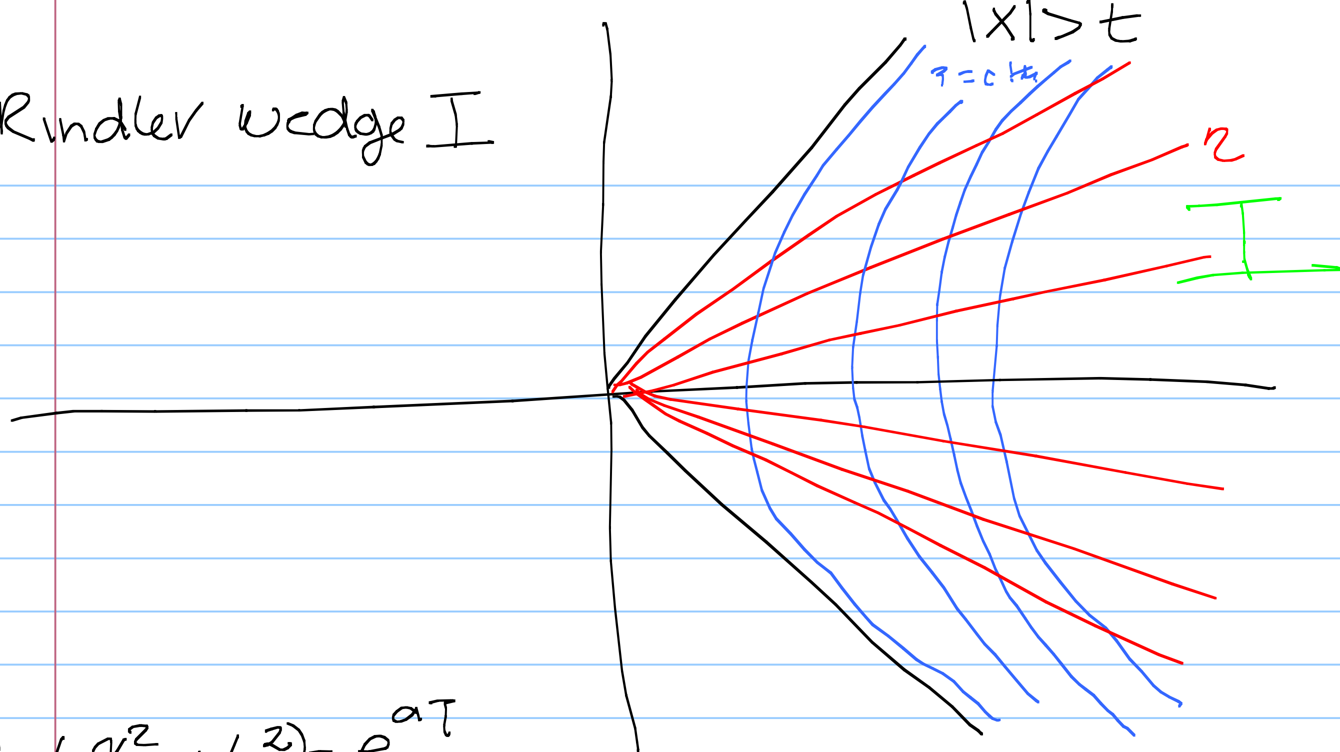
From the perspective of an observer in uniform acceleration

We define a new coordinate system for these observers: Rindler Coordinates (η, ξ)

$$X = \frac{e^{a\xi}}{a}\cosh \eta a \qquad a \text{ is a cte acceleration}$$

$$t = \frac{e^{a\xi}}{a}\sinh \eta a$$

Rindler wedge I



$$a(x^2 - t^2) = e^{a\xi}$$

$$\frac{x}{t} = \tanh a\eta \qquad x = t \qquad \eta \to \infty$$

where the line element in Rindler coordinates is

$$ds^2 = (d\eta^2 - dx^2)\,e^{2a\xi}$$

The Klein-Gordon equation is

$$\left(\frac{\partial^2}{\partial \eta^2} + \frac{\partial^2}{\partial \xi^2}\right)\phi = 0$$

with solutions

$$u_k^{I} = \frac{e^{ik\xi - i\omega\eta}}{\sqrt{2\pi\omega}} \qquad \omega = |k|, \quad -\infty < k < \infty$$

$$u_k^{*I} = \qquad \text{for } |x| > t$$

We have creation and annihilation operators

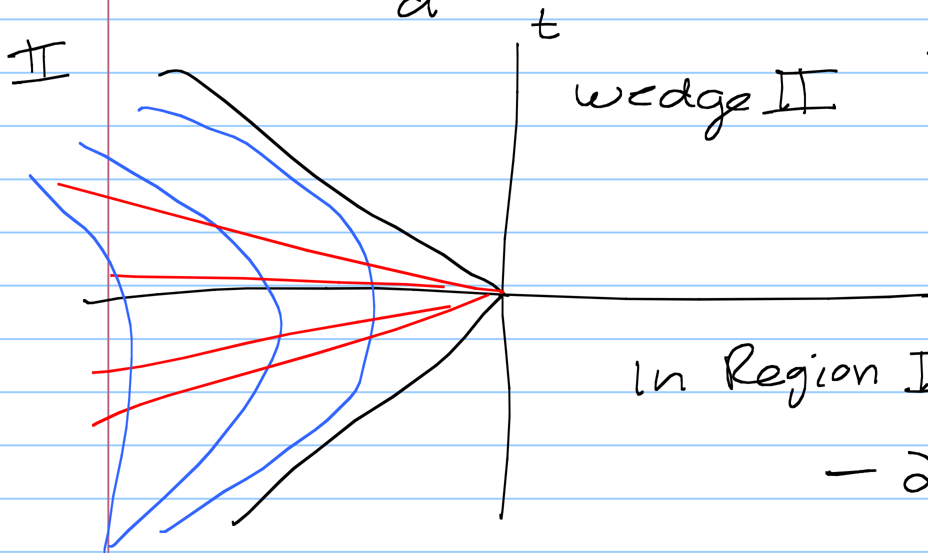
$$a_k^{I} \qquad a_k^{I\dagger} \qquad a_k^{I}|0\rangle_I = 0$$

Fro

$$x = -\frac{e^{a\xi}}{a}\cosh a\eta$$

$$t = -\frac{e^{a\xi}}{a}\sinh a\eta$$

with line element $ds^2 = e^{a\xi}(d\eta^2 - d\xi^2)$



same Klein-Gordon equation

∂_η Killing vector field in Region I

In Region II it is

$-\partial_\eta$

The positive and negative solutions are now

$$\bar{u}_k^{II} = \frac{1}{\sqrt{2\pi w}} e^{ik\xi + iw\eta}$$

$$\bar{u}_k^{*II}$$

a_k^{II}, $a_k^{II\dagger}$

$a_k^{II}|0\rangle_{II} = 0$

The solutions $u_k^{I}, u_k^{II}, u_k^{I*}$ and u_k^{II*} together are a complete set

$$\phi = \sum a_k^I u_k^{I*} + a_k^{II} u_k^{II*} + h.c$$

Vacuum in Rindler coordinates is $|0\rangle_R = |0\rangle_I \otimes |0\rangle_{II}$

what is the relationship ~~wa~~ between $|0\rangle^M$ and the states in Rindler space

$$\left(u_{k'}^*, \sum_k a_k u_k^* + a_k^\dagger u_k\right) = \left(u_{k'}, \sum_k a_k^I u_k^{I*} + a_k^{II} u_k^{II} + h.c.\right)$$

$(u_k, u_{k'}^{I*}) = \alpha_{kk'}$

$$(u_k, u_{k'}^{I*}) = \int (u_k \partial_t u_{k'}^{I*} - \partial_t u_k \partial u_{k'}^{I}) dx$$

$$a_k = \cosh r\, a_k^{I} - \sinh r\, a_k^{II+}$$

where $\cosh r = \dfrac{1}{\sqrt{1 - e^{2\pi w/a}}}$

$a_k |0\rangle^M = 0 \quad |0\rangle^M = \sum A_n |n\rangle_I |n\rangle_{II}$

$$(\cosh r\, a_k^{I} - \sinh r\, a_k^{II+}) \sum A_n |n\rangle_I |n\rangle_{II}$$

$$\sum_n \cosh r\, A_n\, a_k^{I} |n\rangle_I |n\rangle_{II} -$$

$$\sum_n \sinh r\, A_n |n\rangle_I\, a_k^{II+} |n\rangle_{II}$$

$$A_{n+1} = \tanh r\, A_n$$

$$A_n = \tanh^n r\, A_0$$

normalizing the state $\sum_n \tanh^n r\, A_0 |n\rangle_I |n\rangle_{II}$
we find that $A_0 = \dfrac{1}{\cosh r}$

Rob has no acces to Region II

$$|0\rangle^M = \frac{1}{\cosh r} \sum_n \tanh^n r\, |n\rangle_I |n\rangle_{II}$$

two mode squeezed state

Rob must trace over region II

$|0\rangle\langle 0|^{MM} = \sum C_n |n\rangle_I \langle n|_I$ mixed state

C_n are a thermal distribution

$T \sim T(a)$

temperature

$$\frac{1}{\sqrt{2}}\left(|0\rangle_k |0\rangle_{k'} + |1\rangle_k |1\rangle_{k'} \right)$$

$$|1\rangle^{M}_{k'} = a^{+}_{k'} |0\rangle^{M} = \frac{1}{\cosh^2 r} \sum \tanh^n r \sqrt{n+1}\, |n+1\rangle_I |n\rangle_{II}$$

$$|\psi\rangle = \frac{1}{2\cosh r}\left(\sum \tanh^n r\, |0\rangle_A |n\rangle_I |n\rangle_{II} + \frac{1}{\cosh r} \sum \tanh^n r\, |1\rangle |n+1\rangle |n\rangle \right.$$

$$\rho_{AR} = \rho_{A,I}$$