Long-distance Entanglement between two Probes inserted in XX chain

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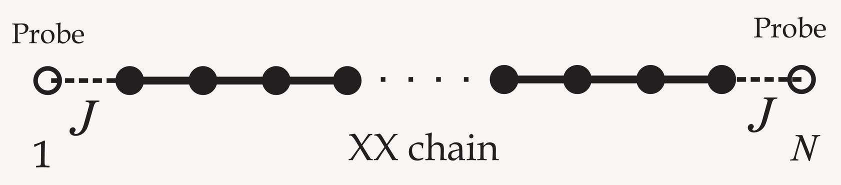
Our Goal

To study ground state entanglement between two probes in this model:



$$\mathcal{H} = \sum_{i=2}^{N-2} \left(S_i^x S_{i+1}^x + S_i^y S_{i+1}^y\right) + J\left(S_1^x S_2^x + S_1^y S_2^y + S_{N-1}^x S_N^x + S_{N-1}^y S_N^y\right)$$

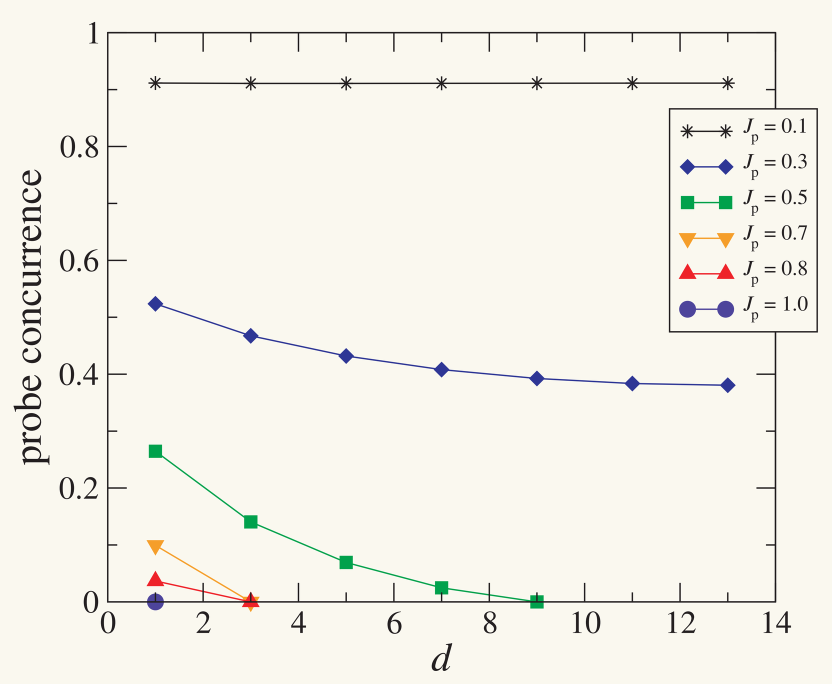
$S^i \equiv \frac{1}{2}\sigma^i$ and N is the number of sites.

- **Why XY model?**
  Because it's exactly solvable.
- **Why *isotropic* XY model?**
  It's hamiltonian commutes with total spin-z operator and the reduced density matrix of probes becomes simple.
- **Why are the probes placed at the ends of chain?**
  In other places, after Jordan-Wigner transformation, produce interacting fermions.

Others Work

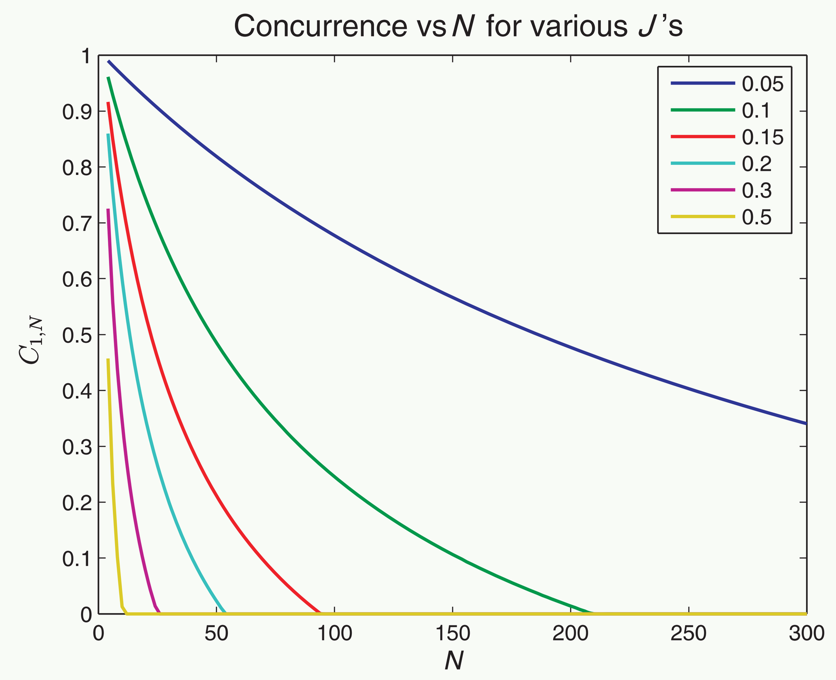
L.C. Venuti, C.D. Esposti Boschi, M. Roncaglia, Phys. Rev. Lett. **96**, 247206 (2006).

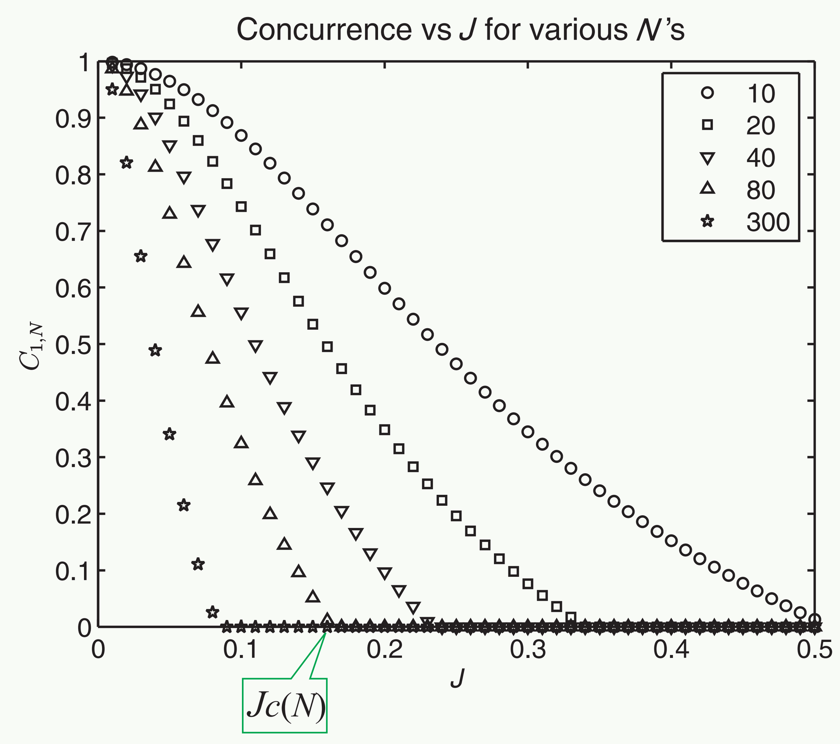
They studied entanglement of two probes inserted in Heisenberg model, numerically (DMRG). Their result is:

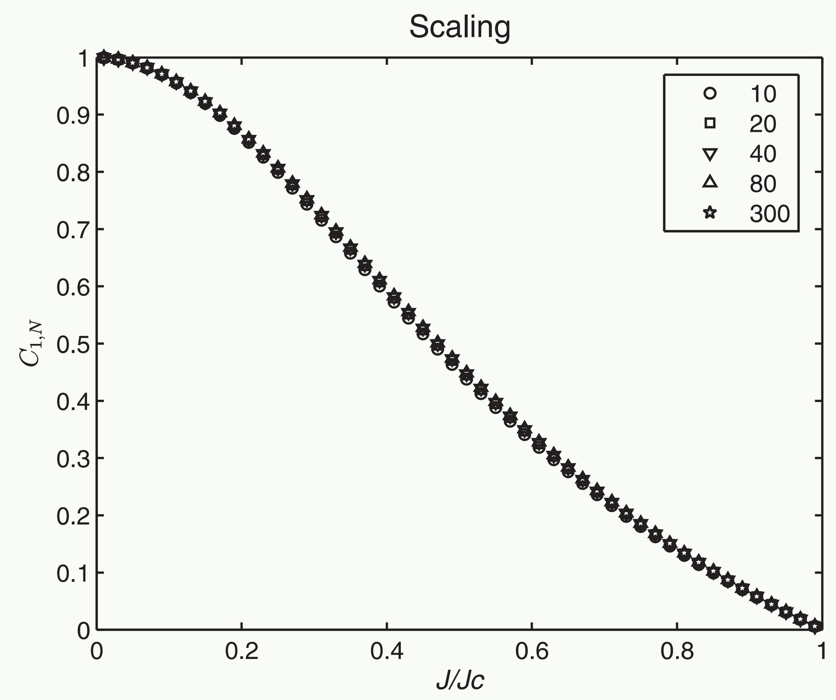


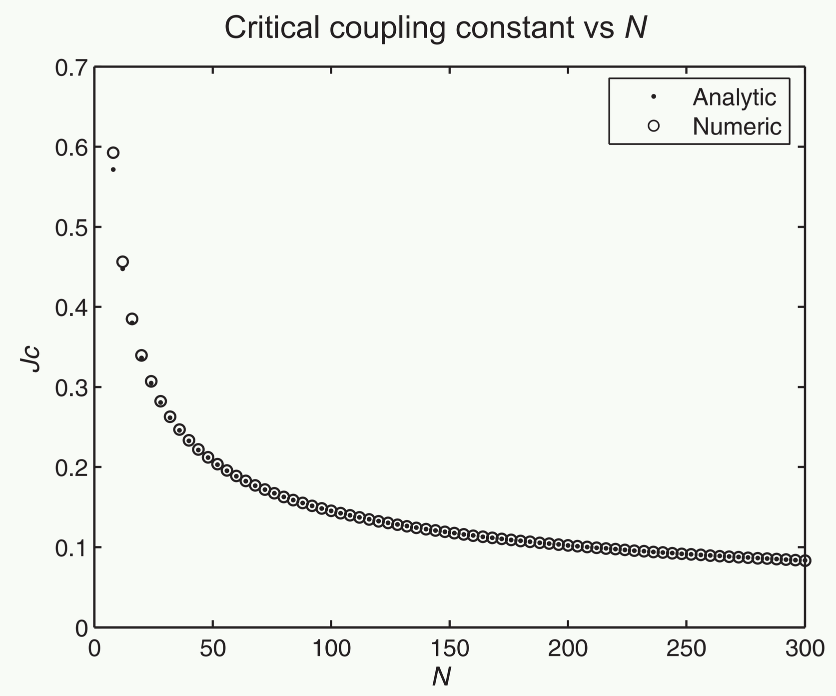
In this graph, d is the distance between probes in the ring and J_p is the coupling constant of probes with the ring.

Our Results









$$C_{1,N}(J,N) = C\left(\frac{J}{J_c(N)}\right) \qquad \left.\frac{\partial C_{1,N}}{\partial J}\right|_{J_c} = \frac{C'(1)}{J_c}$$

J_c for large N's

$$J_c \propto \frac{1}{\sqrt{N}}$$

Our Method

The hamiltonian after Jordan-Wigner transformation

$$\mathcal{H} = \frac{1}{2}\sum_{i=2}^{N-2}\left(c_i^\dagger c_{i+1} + c_{i+1}^\dagger c_i\right) + \frac{J}{2}\left(c_1^\dagger c_2 + c_2^\dagger c_1 + c_{N-1}^\dagger c_N + c_N^\dagger c_{N-1}\right)$$

$$\{c_i, c_j^\dagger\} = \delta_{ij},\ \{c_i, c_j\} = 0$$

$$\mathcal{H} = \sum_{i,j} c_i^\dagger H_{ij} c_j$$

$$H = \frac{1}{2}\begin{pmatrix} 0 & J & & & & \\ J & 0 & 1 & & & \\ & 1 & 0 & & & \\ & & & \ddots & & \\ & & & & & 1 \\ & & & & 1 & 0 & J \\ & & & & & J & 0 \end{pmatrix}_{N\times N}$$

Diagonalization

$$b_k = \sum_i \phi_{ki} c_i, \qquad \mathcal{H} = \sum_k \lambda_k b_k^\dagger b_k$$

$$\sum_k \phi_{ki}\phi_{kj} = \delta_{ij}$$

Concurrence of probes

$$G_{ij} \equiv \sum_{\lambda_k<0} \phi_{ki}\phi_{kj} - \sum_{\lambda_k>0} \phi_{ki}\phi_{kj}$$

$$C_{1,N} = \max\left\{0, |G_{1N}| + \frac{1}{2}G_{1N}^2 - \frac{1}{2}\right\}$$

References

E. Lieb, T. Schultz, D. Mattis, Ann. Phys. **60**, 407 (1961).

U. Glaser, H. Büttner, H. Fehske, Phys. Rev. A **68**, 032318 (2003).