Universal Holonomic Quantum Computation on Spin Chains

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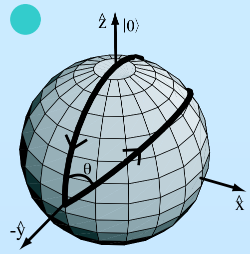


We find exact solution for a universal set of holonomic quantum gates on a scalable candidate for quantum computers , namely an array of two level systems. These gates have been constructed on a scalable systems without any numerical search in the space of control parameters of the Hamiltonian.

1- The holonomy associated with the loop $P(t)=e^{tX}P_0e^{-tX}$

or the family of iso- spectral hamiltonians $H(t)=e^{tX}H_0e^{-tX}$ **:**

$$U_{gate}=Pe^{\oint_C A}=e^A\in U(k)$$

$$A=X\big|_k\ ,\ H_0=\varepsilon\sum_{i=1}^{k}|i\rangle\langle i|\equiv\varepsilon P_0\ ,\quad t\in[0,1]\ .$$

The exact solution of the inverse problem to reproduce U_{gate} :

1-To have non trivial solution: $[X,P_0]\neq 0$.

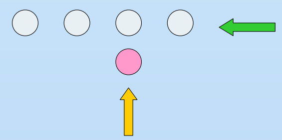
2-To have a closed loop: $[e^X,P_0]=0$.

3-To have our desirable gate: $U_{gate}=e^A$.

The general form of the U_{gate} contains an inevitable dynamical phase:

$$U_{gate}=e^{-i\varepsilon}e^A\ .$$

- **Physical realization*:**



2- Holonomic Quantum computing on a spin chain:

1- Take an array of q- bits with the following Heisenberg interaction: $H_0=B(\sigma_{1z}+\sigma_{2z})+J\vec{\sigma}_1.\vec{\sigma}_2$.

2- Choose the magnetic field so that B= 2J.

3- Take the codes or the computational bases to be the degenerate ground states:

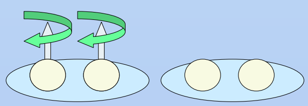
$$|0\rangle\equiv|\phi_0\rangle=|-,-\rangle\ ,\ |1\rangle\equiv|\psi_0\rangle=\tfrac{1}{\sqrt{2}}(|+,-\rangle-|-,+\rangle)\ .$$

4- Take the operator X to be: $X=i\vec{n}.(\omega_1\vec{\sigma}_1+\omega_2\vec{\sigma}_2)$.

After the lapse of time T= 1 and acquiring the dynamical phase 3J, at the end of any Loop we can stop and only pause for a time interval τ . This lapse of time will add a phase $3J\tau$ to the above phase and then we obtain a general unitary gate given by:

$$U_{gate}=e^{i\vec{r}.\vec{\sigma}+r_z+3J(1+\tau)}\ .$$

A. The Phase Gate:



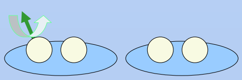
rotate each spin in the block in this way:

1- Take z-axis the axis of rotation.

2- Rotate each spin with the same frequency which equal to $-\frac{\phi}{2}$.

3- Pause after each loop so that τ satisfy $3J(1+\tau)=\phi+2m\pi$.

B. The Hadamard Gate:



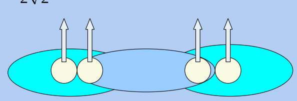
rotate each spin in the block in this way:

1- Take $\vec{n}=(\sqrt{\tfrac{1}{3}},0,-\sqrt{\tfrac{2}{3}})$ the axis of rotation.

2- Rotate only one spin with the frequency which equal to $\omega=\frac{\pi}{2}\sqrt{3}$.

3- Pause after each loop so that τ satisfy $3J(1+\tau)=-\frac{\pi}{2\sqrt{2}}(\sqrt{2}+1)+2m\pi$.

C. The Conditional Phase Gate:



1- Take X in the following way: $X=i\phi(\sigma_{2z}\sigma_{3z}+\sigma_{2z}+\sigma_{3z})$.

2- Pause after each loop so that τ satisfy $6J(1+\tau)=\phi+2m\pi$.

References: 1- S. Tanimura, D. Hayashi, M. nakahara, Phys. Lett. A 325, 199 (2004).

2- V. Karimipour, N. Majd, Phys. Rev. A 72. 052305 (2005).

***- V. Karimipour, N. Majd, Phys. Rev. A 70. 012320 (2004).**