Nonlocality for graph states

Adán Cabello[1], Otfried Gühne[2], Pilar Moreno[1], and David Rodríguez[1].

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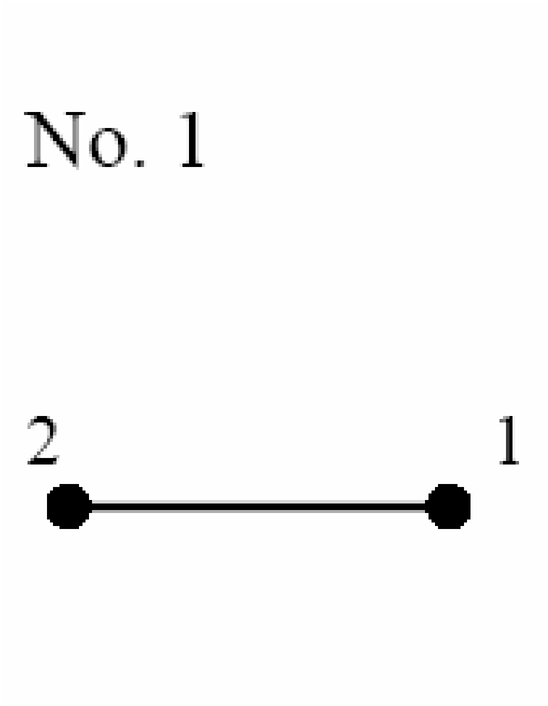
1st International Iran Conference on Quantum Information (Kish Island, Iran),

September 9th, 2007.

Plan

- Graph states
- EPR, Bell and the loophole-free experiment
- GHZ and exponentially growing with size nonlocality
- Bipartite AVN and the loophole-free experiment
- Optimal Bell inequalities for graph states

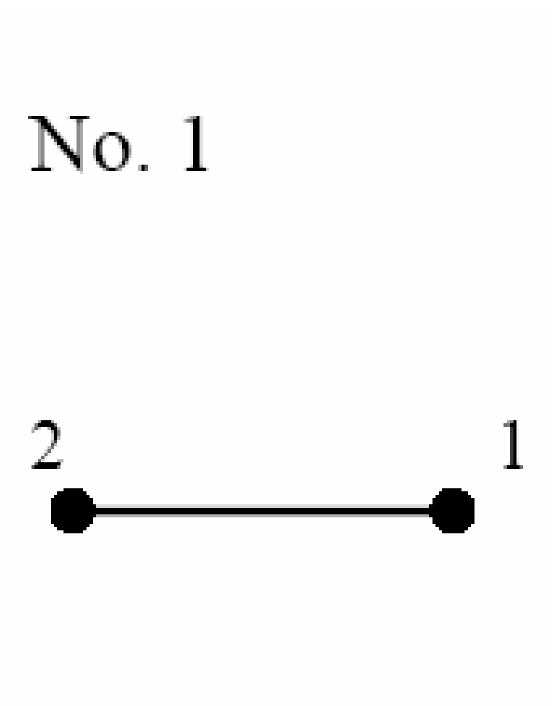
Graph states, examples

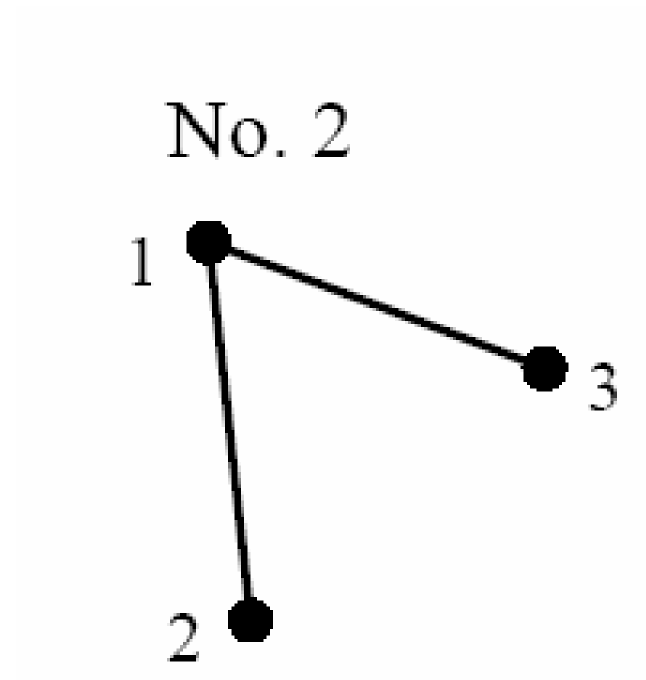


Eq. associated to qubit 1: $XZ\,|\phi_1\rangle = |\phi_1\rangle$,

“ “ “ “ 2: $ZX\,|\phi_1\rangle = |\phi_1\rangle$.

Graph states, examples





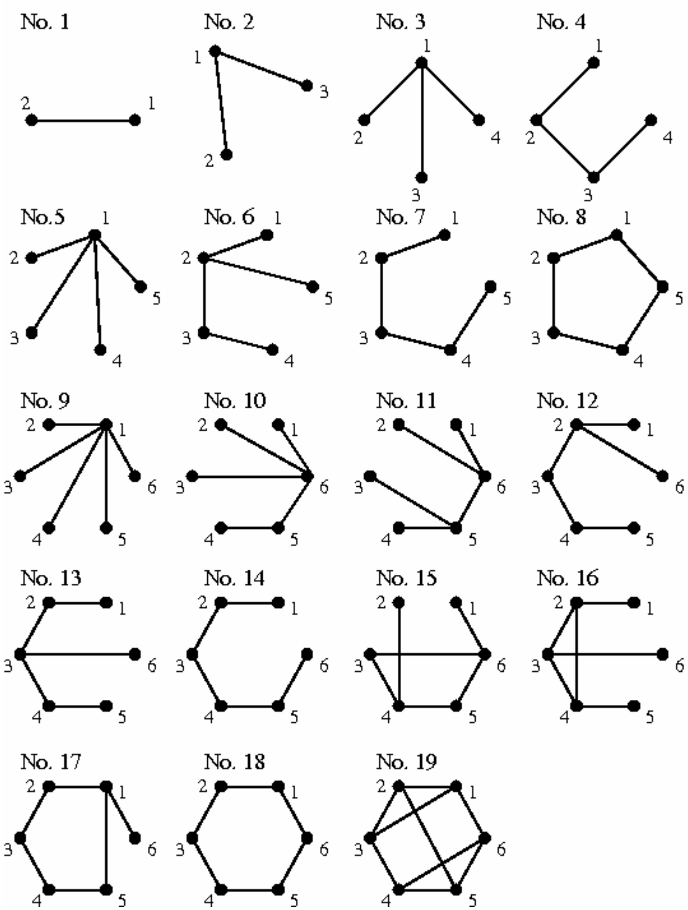
Eq. associated to qubit 1: $XZ\,|\phi_1\rangle = |\phi_1\rangle$,
" " " " 2: $ZX\,|\phi_1\rangle = |\phi_1\rangle$.

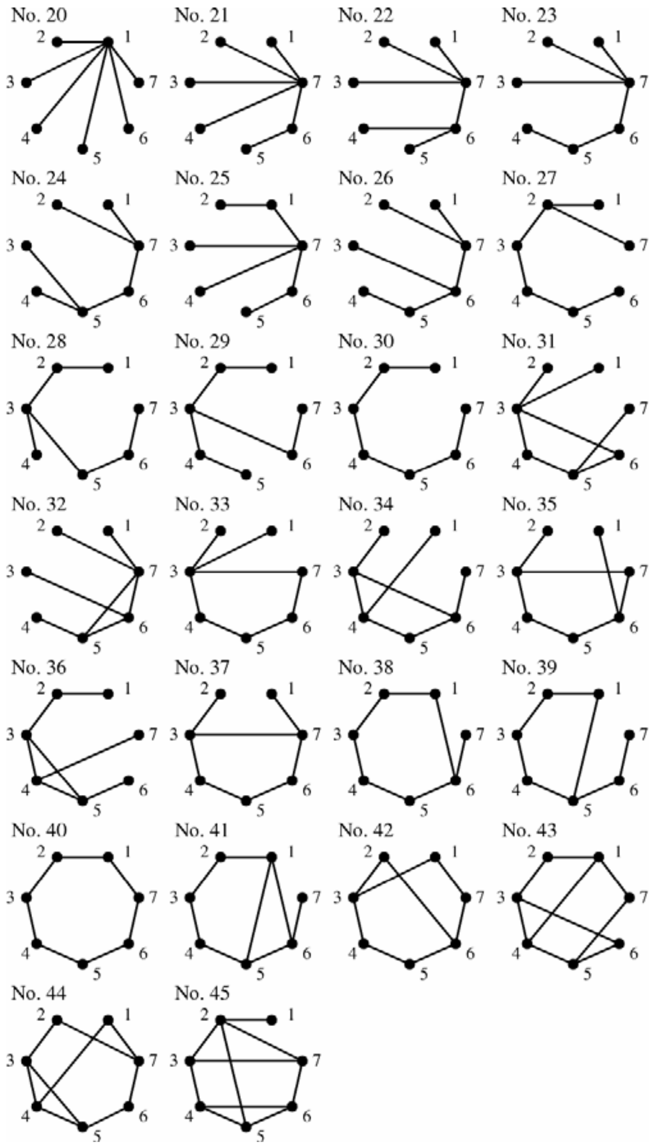
" " " " 1: $XZZ\,|\phi_2\rangle = |\phi_2\rangle$,
" " " " 2: $ZXI\,|\phi_2\rangle = |\phi_2\rangle$,
" " " " 3: $ZIX\,|\phi_2\rangle = |\phi_2\rangle$.

Graph states are useful

- All-versus-nothing (AVN) nonlocality proofs
- Exponentially-growing-with size nonlocality
- Quantum error correction
- Classification of entanglement
- Initial state for measurement-based quantum computation

All graph states up to seven qubits





Graphs corresponding to all possible graph states, up to seven qubits, which are not equivalent under single-qubit transformations and graph isomorphism, M. Hein, J. Eisert, and H. J. Briegel, Phys. Rev. A **69**, 062311 (2004).

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The first paper on “quantum nonlocality”

THE NEW YORK TIMES, SATURDAY, MAY 4, 1935.

EINSTEIN ATTACKS QUANTUM THEORY

Scientist and Two Colleagues Find It Is Not 'Complete' Even Though 'Correct.'

SEE FULLER ONE POSSIBLE

Believe a Whole Description of 'the Physical Reality' Can Be Provided Eventually.

Copyright 1935 by Science Service.

PRINCETON, N. J., May 3.—Professor Albert Einstein will attack science's important theory of quantum mechanics, a theory of which he was a sort of grandfather. He concludes that while it is "correct" it is not "complete."

With two colleagues at the Institute for Advanced Study here, the noted scientist is about to report to the American Physical Society what is wrong with the theory of quantum mechanics, it has been learned exclusively by Science Service.

The quantum theory, with which science predicts with some success inter-atomic happenings, does not meet the requirements for a satisfactory physical theory. Professor Einstein will report in a joint paper with Dr. Boris Podolsky and Dr. N. Rosen.

In the quantum theory as now used, the latest Einstein paper will point out that where two physical quantities such as the position of a particle and its velocity interact, a knowledge of one quantity precludes knowledge about the other. This is the famous principle of uncertainty put forward by Professor Werner Heisenberg and incorporated in the quantum theory. This very fact, Professor Einstein feels, makes the quantum theory fail in the requirements necessary for a satisfactory physical theory.

Two Requirements Listed.

These two requirements are:

1. The theory should make possible a calculation of the facts of nature and predict results which can be accurately checked by experiment; the theory should be, in other words, correct.

2. Moreover, a satisfactory theory should, as a good image of the objective world, contain a counterpart for things found in the objective world; that is, it must be a complete theory.

Quantum theory, Professor Einstein and his colleagues will report, fulfills the correctness requirement but fails in the completeness requirement.

While proving that present quantum theory does not give a complete description of physical reality, Professor Einstein believes some later, still undeveloped, theory will make this possible. His conclusion is:

"While we have thus shown that the wave function [of quantum theory] does not provide a complete description of the physical reality, we left open the question of whether or not such a description exists. We believe, however, that such a theory is possible."

The development of quantum mechanics has proved very useful in exploring the atom. Six Nobel Prizes in physics, including one to Einstein, have been awarded for various phases of the researches leading up to quantum mechanics. The names of Planck, Bohr, de Broglie, Heisenberg, Dirac and Schroedinger, as well as Einstein, are linked with quantum mechanics.

The exact title of the Einstein-Podolsky-Rosen paper is: "Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?"

Explanation by Podolsky.

In explaining the latest view of the physical world as revealed in their researches Dr. Podolsky, one of the authors, said:

"Physicists believe that there exist real material things independent of our minds and our theories. We construct theories and invent words (such as electron, positron, &c.) in an attempt to explain to ourselves what we know about our external world and to help us to obtain further knowledge of it. Before a theory can be considered to be satisfactory it must pass two very severe tests. First, the theory must enable us to calculate facts of nature, and these calculations must agree very accurately with observations and experiments. Second, we expect a satisfactory theory, as a good image of objective reality, to contain a counterpart for every element of the physical world. A theory satisfying the first requirement may be called a correct theory while, if it satisfies the second requirement, it may be called a complete theory.

"Hundreds of thousands of experiments and measurements have shown that, at least in cases when matter moves much slower than light, the theory of Planck, Einstein, Bohr Heisenberg and Schroedinger known as quantum mechanics is a correct theory. Einstein, Podolsky and Rosen now discuss the question of the completeness of quantum mechanics. They arrive at the conclusion that quantum mechanics, in its present form, is not complete.

"In quantum mechanics the condition of any physical system, such as an electron, an atom, &c., is supposed to be completely described by a formula known as a 'wave function.' Suppose that we know the wave function for each of two physical systems, and that these two systems come together, interact, and again separate (as when two particles collide and move apart). Quantum mechanics, although giving us considerable information about such a process, does not enable us to calculate the wave function of each physical system after the separation. This fact is made use of in showing that the wave function does not give a complete description of physical reality. Since, however, description of physical systems by wave functions is an essential step of quantum mechanics, this means that quantum mechanics is not a complete theory."

Raises Point of Doubt.

Special to THE NEW YORK TIMES.

PRINCETON, N. J., May 3.—Asked to comment on the new ideas of Professor Einstein and his collaborators, Professor Edward U. Condon, mathematical physicist of Princeton University, said tonight:

"Of course, a great deal of the argument hinges on just what meaning is to be attached to the word 'reality' in connection with physics. They have certainly discussed an interesting point in connection with the theory. Dr. Einstein has never been satisfied with the statistical causality which in the new theories replaces the strict causality of the old physics.

"It is reported that when he first learned of the work of Schroedinger and Dirac, he said, 'Der lieber Gott wuerfelt nicht, [the good Lord does not throw dice]. For the last five years he has subjected the quantum mechanical theories to very searching criticism from this standpoint. But I am afraid that thus far the statistical theories have withstood criticism."

EPR: QM is *not* complete

MAY 15, 1935 PHYSICAL REVIEW VOLUME 47

Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. Einstein, B. Podolsky and N. Rosen, *Institute for Advanced Study, Princeton, New Jersey*

(Received March 25, 1935)

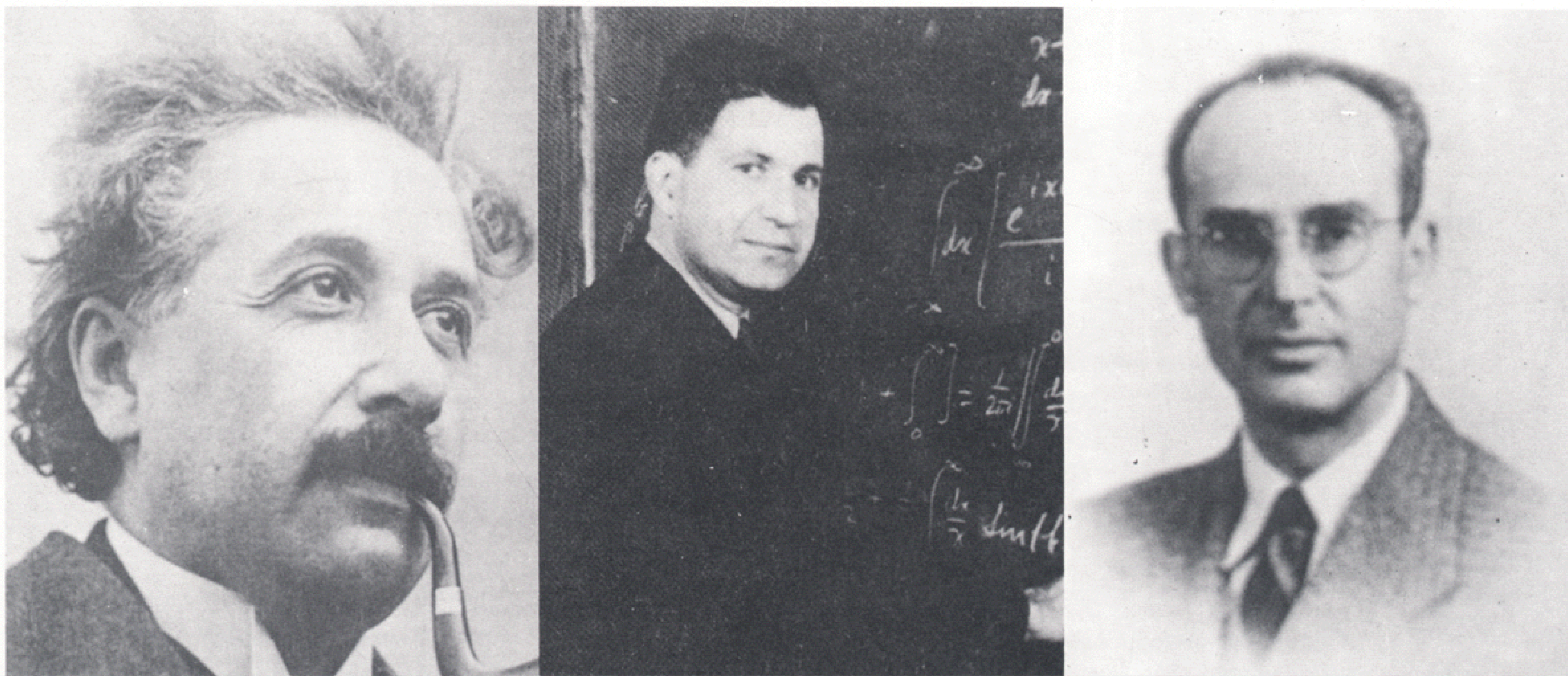
In a complete theory there is an element corresponding to each element of reality. A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system. In quantum mechanics in the case of two physical quantities described by non-commuting operators, the knowledge of one precludes the knowledge of the other. Then either (1) the description of reality given by the wave function in quantum mechanics is not complete or (2) these two quantities cannot have simultaneous reality. Consideration of the problem of making predictions concerning a system on the basis of measurements made on another system that had previously interacted with it leads to the result that if (1) is false then (2) is also false. One is thus led to conclude that the description of reality as given by a wave function is not complete.

According to EPR, any satisfactory physical theory must be:

(1) Correct.

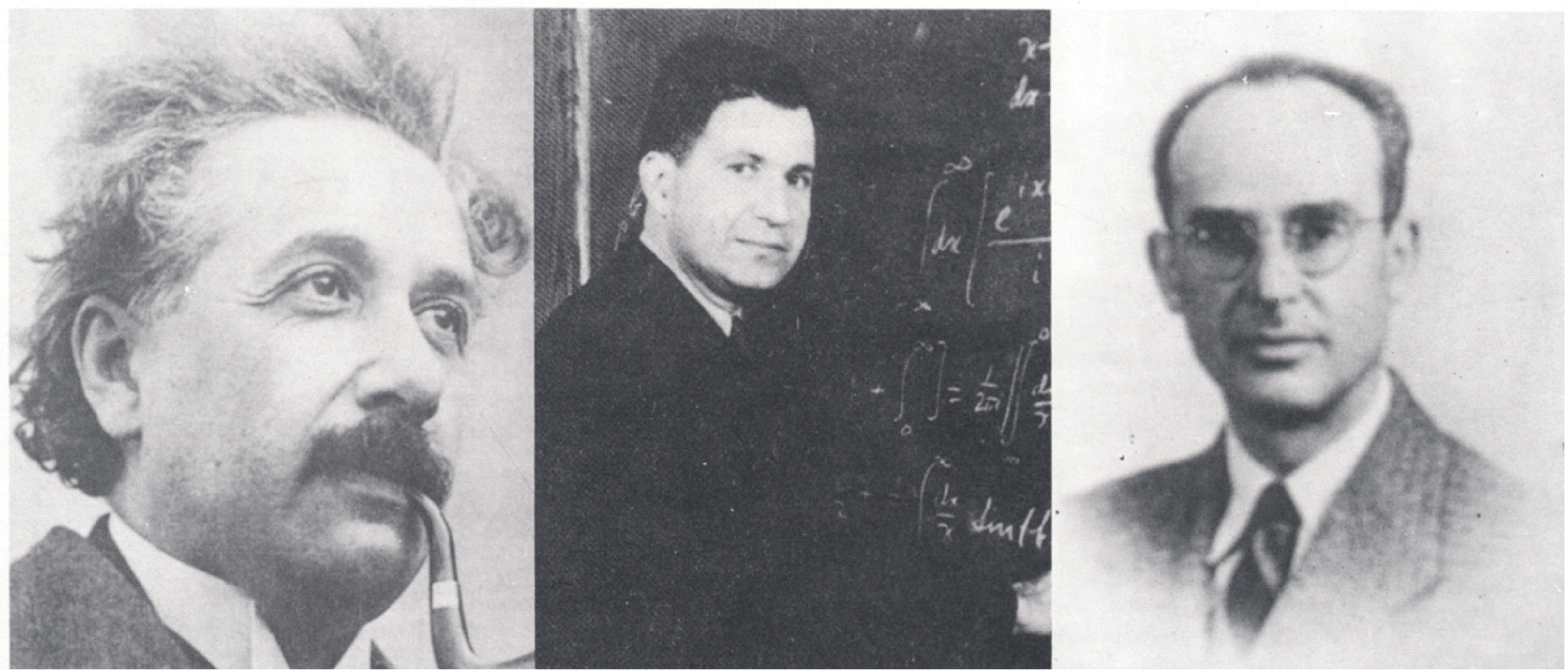
(2) “Complete”.

EPR’s elements of reality



“If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.”

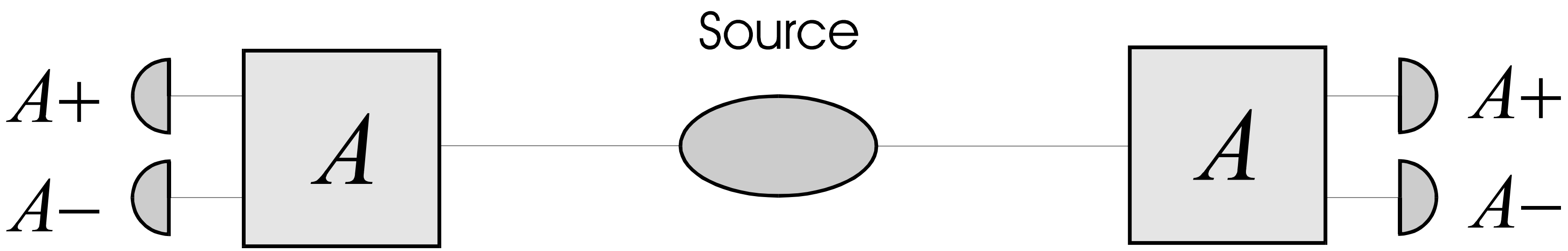
EPR’s elements of reality



“Without in any way disturbing a system” = Spacelike separation.

“Predict with certainty” = Perfect correlations.

Bohm's version of EPR's argument



$$\left|\psi^{-}\right\rangle = \frac{1}{\sqrt{2}}\left(\left|01\right\rangle - \left|10\right\rangle\right)$$

Bohm's version of EPR's argument

$$X_1 X_2 = -1$$

$$Y_1 Y_2 = -1$$

- X_2 and Y_2 are both "elements of reality".

- In QM, X_2 and Y_2 are incompatible observables (Heisenberg's uncertainty principle).

→ *QM is incomplete (according to EPR).*

Bell's theorem

It is impossible to complete QM with elements of reality because some predictions of QM cannot be reproduced with elements of reality.

The problem of a loophole-free Bell experiment

So far, the results of any performed Bell experiment admit an interpretation in terms of local realistic theories.

A loophole-free experiment would require:

- Spacelike separation between Alice’s measurement choice and Bob’s measurement in order to exclude the possibility that Alice's measurement choice influences the result of Bob's measurement (locality loophole).

- Sufficiently large number of detections of the prepared particles in order to exclude the possibility that the nondetections correspond to local hidden-variable instructions (detection loophole).

Problem

43 years after Bell's original paper we do not have a loophole-free Bell experiment!

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Greenberger, Horne and Zeilinger

GHZ's proof of Bell's theorem

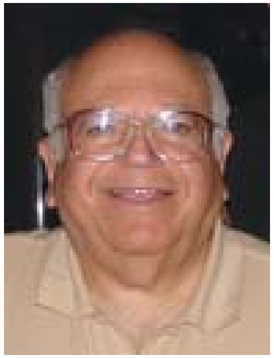
- "Opened a new chapter on the hidden variables problem"", and made "the strongest case against local realism since Bell's work".

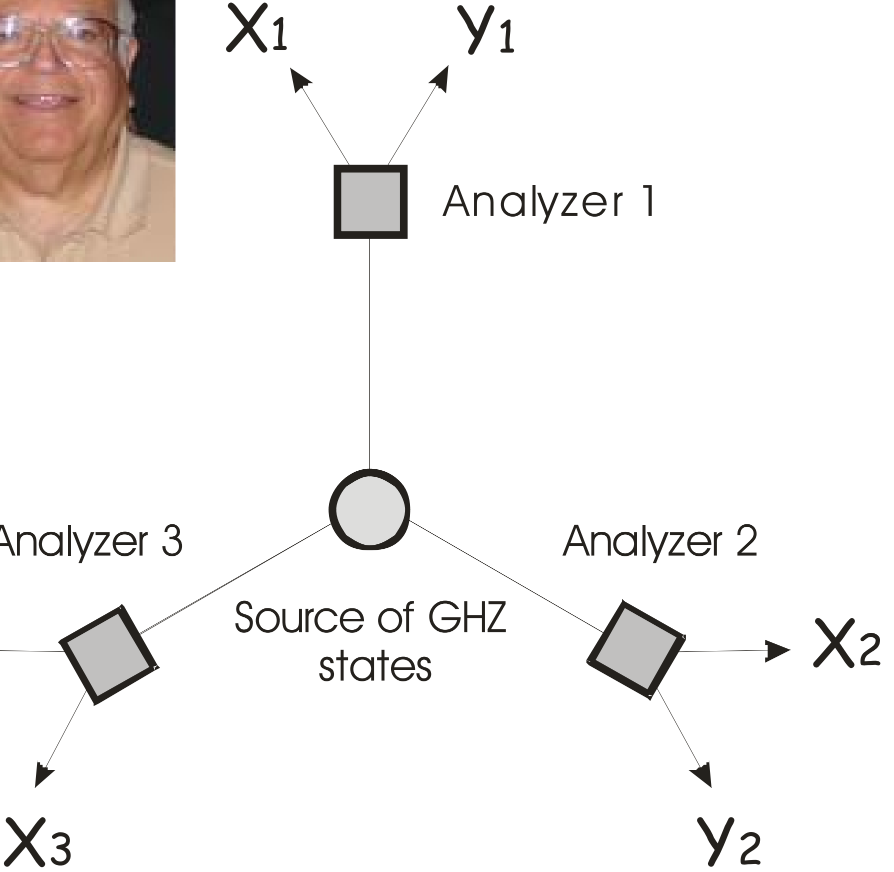
- Quantum reduction of the communication complexity.

- Quantum secret sharing.

- Multipartite entanglement.

GHZ’s proof of Bell’s theorem





GHZ's proof of Bell's theorem

$$|GHZ\rangle = |HHH\rangle - |VVV\rangle$$

$$X_1 Y_2 Y_3 |GHZ\rangle = |GHZ\rangle$$

$$Y_1 X_2 Y_3 |GHZ\rangle = |GHZ\rangle$$

$$Y_1 Y_2 X_3 |GHZ\rangle = |GHZ\rangle$$

$$X_1 X_2 X_3 |GHZ\rangle = -|GHZ\rangle$$

Notation for single photon observables

Polarization observables:

$$X = |H\rangle\langle V| + |V\rangle\langle H|$$

$$Y = i\,(|V\rangle\langle H| - |H\rangle\langle V|)$$

$$Z = |H\rangle\langle H| - |V\rangle\langle V|$$

GHZ's proof of Bell's theorem: X_i and Y_i are ER

GHZ:

$$|HHH\rangle - |VVV\rangle$$

$$v(X_1)v(Y_2) = v(Y_3)$$

$$v(Y_1)v(X_2) = v(Y_3)$$

$$v(Y_1)v(Y_2) = v(X_3)$$

$$v(X_1)v(X_2) = -\,v(X_3)$$

GHZ's proof of Bell's theorem: Contradiction!

GHZ:

$$|HHH\rangle - |VVV\rangle$$

$$v(X_1)v(Y_2) = v(Y_3)$$

$$v(Y_1)v(X_2) = v(Y_3)$$

$$v(Y_1)v(Y_2) = v(X_3)$$

$$v(X_1)v(X_2) = -\,v(X_3)$$

Why "all-versus-nothing"?

$$v(X_1)v(Y_2) = v(Y_3)$$

$$v(Y_1)v(X_2) = v(Y_3)$$

$$v(Y_1)v(Y_2) = v(X_3)$$

Why “all-versus-nothing”?

$$v(X_1)v(Y_2) = v(Y_3)$$

$$v(Y_1)v(X_2) = v(Y_3)$$

$$v(Y_1)v(Y_2) = v(X_3)$$

$$v(X_1)v(X_2) = +v(X_3)$$

Why “all-versus-nothing”?

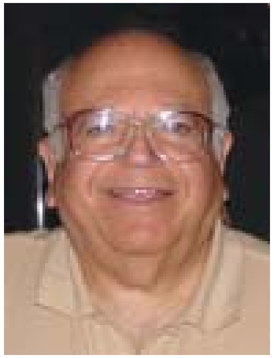
$$v(X_1)v(Y_2) = v(Y_3)$$

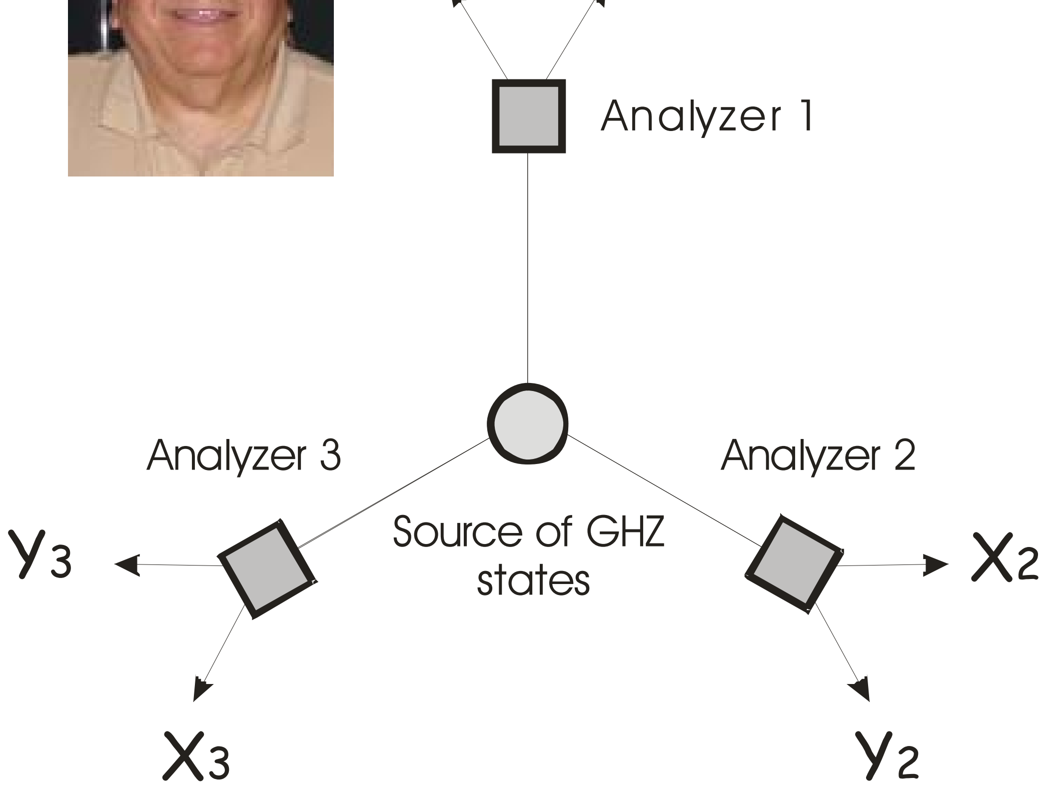
$$v(Y_1)v(X_2) = v(Y_3)$$

$$v(Y_1)v(Y_2) = v(X_3)$$

$$v(X_1)v(X_2) = -v(X_3)$$

GHZ’s requires three observers





Why GHZ's requires three observers?

$$v(X_1)v(Y_2) = v(Y_3)$$

$$v(Y_1)v(X_2) = v(Y_3)$$

$$v(Y_1)v(Y_2) = v(X_3)$$

$$v(X_1)v(X_2) = -v(X_3)$$

Why GHZ's requires three observers?

$$v(X_1)v(Y_2) = v(Y_3)$$

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$$v(Y_1)v(Y_2) = v(X_3)$$

$$v(X_1)v(X_2) = -v(X_3)$$

Nonlocality grows exponentially with size!!!

VOLUME 65, NUMBER 15 PHYSICAL REVIEW LETTERS 8 OCTOBER 1990

Extreme Quantum Entanglement in a Superposition of Macroscopically Distinct States

N. David Mermin

Laboratory of Atomic and Solid State Physics, Cornell University, Ithaca, New York 14853-2501

(Received 29 May 1990)

A Bell inequality is derived for a state of n spin-$\frac{1}{2}$ particles which superposes two macroscopically distinct states. Quantum mechanics violates this inequality by an amount that grows exponentially with n.

Problems for an experimental demonstration

A nonlocality proof using *n*-qubit GHZ states requires *n* space-like separated observers.

For GHZ states decoherence also grows with *n*.

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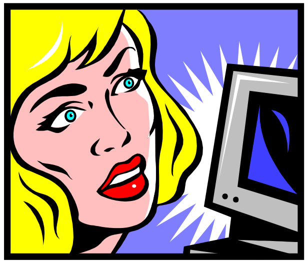
Problem

- Two-observer AVN proofs?



The first two-observer AVN proof

Double Bell:

$$(|HV\rangle - |VH\rangle)\,(|ud\rangle - |du\rangle)$$

$$v(Z_1) = -v(Z_2)$$

$$v(z_1) = -v(z_2)$$

$$v(X_1) = -v(X_2)$$

$$v(x_1) = -v(x_2)$$

$$v(Z_1 z_1) = v(Z_2)\,v(z_2)$$

$$v(X_1 x_1) = v(X_2)\,v(x_2)$$

$$v(Z_1)\,v(x_1) = v(Z_2 x_2)$$

$$v(X_1)\,v(z_1) = v(X_2 z_2)$$

$$v(Z_1 z_1)\,v(X_1 x_1) = -v(Z_2 x_2)\,v(X_2 z_2)$$

Phys. Rev. Lett. **86**, 1911 (2001); **87**, 010403 (2001).

Notation for single photon observables

Polarization observables:

$$X = |H\rangle\langle V| + |V\rangle\langle H|$$

$$Y = i\,(|V\rangle\langle H| - |H\rangle\langle V|)$$

$$Z = |H\rangle\langle H| - |V\rangle\langle V|$$

Path observables:

$$x = |u\rangle\langle d| + |d\rangle\langle u|$$

$$y = i\,(|d\rangle\langle u| - |u\rangle\langle d|)$$

$$z = |u\rangle\langle u| - |d\rangle\langle d|$$

Four qubits in two photons

VOLUME 90, NUMBER 16 PHYSICAL REVIEW LETTERS week ending 25 APRIL 2003

All-Versus-Nothing Violation of Local Realism for Two Entangled Photons

Zeng-Bing Chen,[1] Jian-Wei Pan,[1,2] Yong-De Zhang,[1] Časlav Brukner,[2] and Anton Zeilinger[2]
[1]Department of Modern Physics, University of Science and Technology of China, Hefei, Anhui 230027, China
[2]Institut für Experimentalphysik, Universität Wien, Boltzmanngasse 5, 1090 Wien, Austria
(Received 18 November 2002; published 24 April 2003)

It is shown that the Greenberger-Horne-Zeilinger theorem can be generalized to the case with only two entangled particles. The reasoning makes use of two photons which are maximally entangled both in polarization and in spatial degrees of freedom. In contrast to Cabello's argument of "all versus nothing" nonlocality with four photons [Phys. Rev. Lett. **87**, 010403 (2001)], our proposal to test the theorem can be implemented with linear optics and thus is well within the reach of current experimental technology.

$$z_1 \cdot z_2|\Psi\rangle_{12} = -|\Psi\rangle_{12}, \qquad z_1' \cdot z_2'|\Psi\rangle_{12} = -|\Psi\rangle_{12},$$

$$x_1 \cdot x_2|\Psi\rangle_{12} = -|\Psi\rangle_{12}, \qquad x_1' \cdot x_2'|\Psi\rangle_{12} = -|\Psi\rangle_{12},$$

$$z_1 z_1' \cdot z_2 \cdot z_2'|\Psi\rangle_{12} = |\Psi\rangle_{12},$$

$$x_1 x_1' \cdot x_2 \cdot x_2'|\Psi\rangle_{12} = |\Psi\rangle_{12},$$

$$z_1 \cdot x_1' \cdot z_2 x_2'|\Psi\rangle_{12} = |\Psi\rangle_{12},$$

$$x_1 \cdot z_1' \cdot x_2 z_2'|\Psi\rangle_{12} = |\Psi\rangle_{12},$$

$$z_1 z_1' \cdot x_1 x_1' \cdot z_2 x_2' \cdot x_2 z_2'|\Psi\rangle_{12} = -|\Psi\rangle_{12}.$$

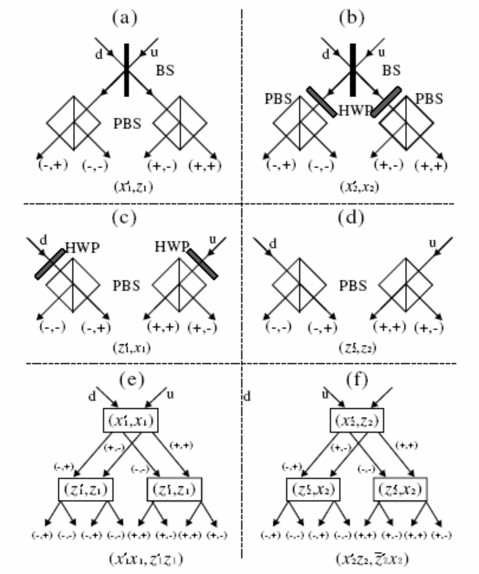


FIG. 2. Six apparatuses for measuring z_1, x_1', and $z_1 \cdot x_1'$ (a); x_2, x_2', and $x_2 \cdot x_2'$ (b); z_1', x_1, and $x_1 \cdot z_1'$ (c); z_2, z_2', and $z_2 \cdot z_2'$ (d); $z_1 z_1'$, $x_1 x_1'$, and $z_1 z_1' \cdot x_1 x_1'$ (e); $z_2 x_2'$, $x_2 z_2'$, and $z_2 x_2' \cdot x_2 z_2'$ (f). By $\pm$, we mean ± 1.

Rome and Hefei experiments

PRL 95, 240405 (2005) PHYSICAL REVIEW LETTERS week ending 9 DECEMBER 2005

All-Versus-Nothing Nonlocality Test of Quantum Mechanics by Two-Photon Hyperentanglement

C. Cinelli, M. Barbieri, R. Perris, P. Mataloni, and F. De Martini

Dipartimento di Fisica dell' Università "La Sapienza"
and Consorzio Nazionale Interuniversitario per le Scienze Fisiche della Materia, Roma, 00185 Italy

(Received 27 April 2005; published 9 December 2005)

We report the experimental realization and the characterization of polarization and momentum hyperentangled two-photon states, generated by a new parametric source of correlated photon pairs. By adoption of these states an "all-versus-nothing" test of quantum mechanics was performed. The two-photon hyperentangled states are expected to find at an increasing rate a widespread application in state engineering and quantum information.

PRL 95, 240406 (2005) PHYSICAL REVIEW LETTERS week ending 9 DECEMBER 2005

All-Versus-Nothing Violation of Local Realism by Two-Photon, Four-Dimensional Entanglement

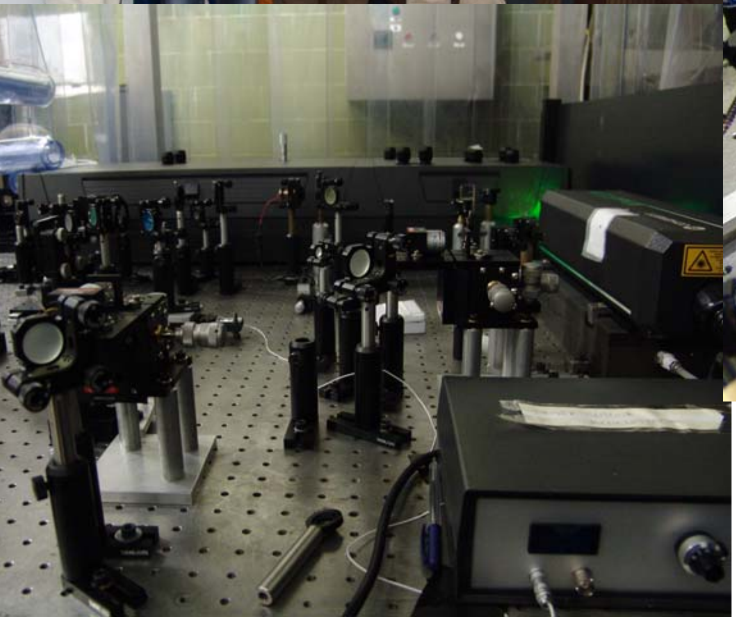
Tao Yang,[1] Qiang Zhang,[1] Jun Zhang,[1] Juan Yin,[1] Zhi Zhao,[1,2] Marek Żukowski,[3]
Zeng-Bing Chen,[1,2,*] and Jian-Wei Pan[1,2,†]

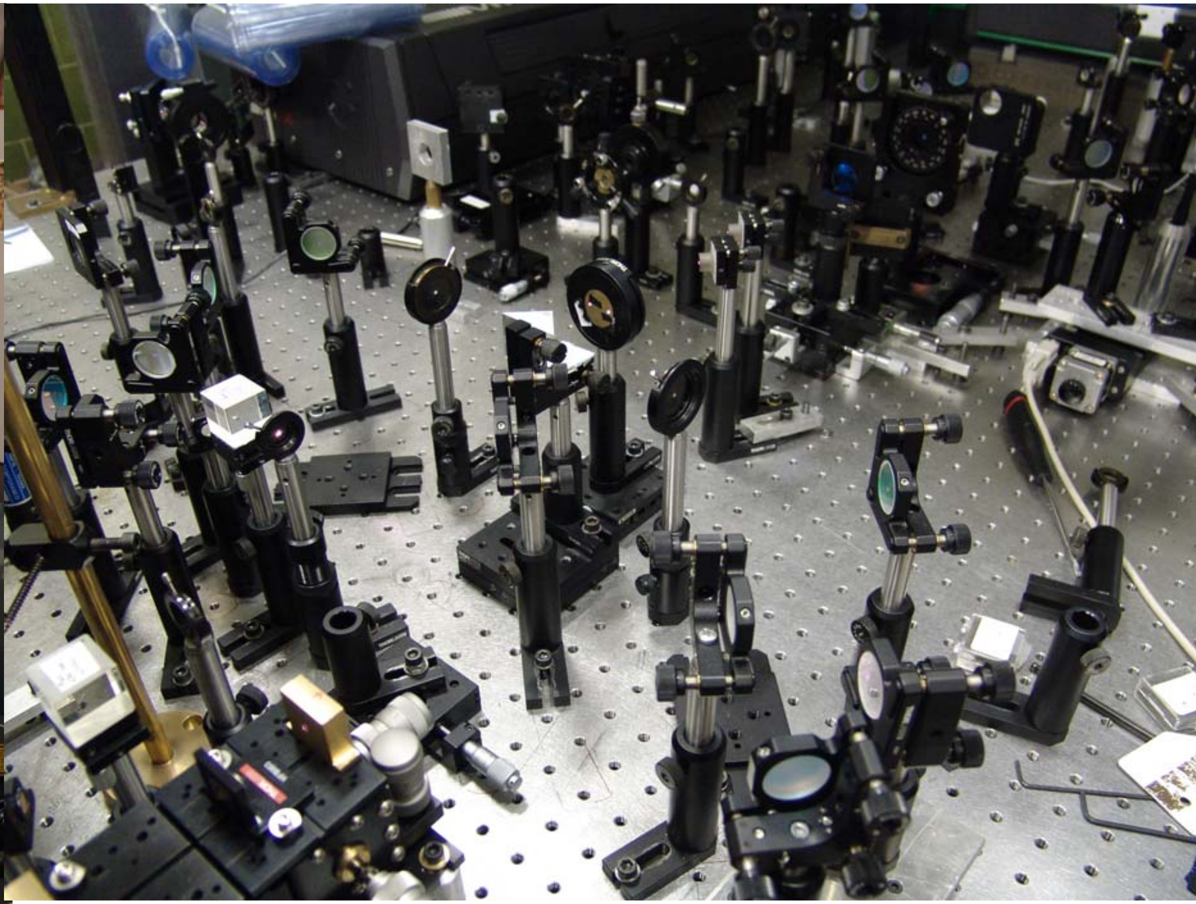
[1]*Hefei National Laboratory for Physical Sciences at Microscale and Department of Modern Physics, University of Science and Technology of China, Hefei, Anhui 230026, China*
[2]*Physikalisches Institut, Universität Heidelberg, Philosophenweg 12, D-69120 Heidelberg, Germany*
[3]*Instytut Fizyki Teoretycznej i Astrofizyki Uniwersytet Gdański, PL-80-952 Gdańsk, Poland*

(Received 4 June 2005; published 9 December 2005)

We develop and exploit a source of two-photon, four-dimensional entanglement to report the first two-particle all-versus-nothing test of local realism with a linear optics setup, but without resorting to a noncontextuality assumption. Our experimental results are in good agreement with quantum mechanics while in extreme contradiction to local realism. Potential applications of our experiment are briefly discussed.

Rome experiment 2005





Rome experiment 2005

All-Versus-Nothing Nonlocality Test of Quantum Mechanics by Two-Photon Hyperentanglement

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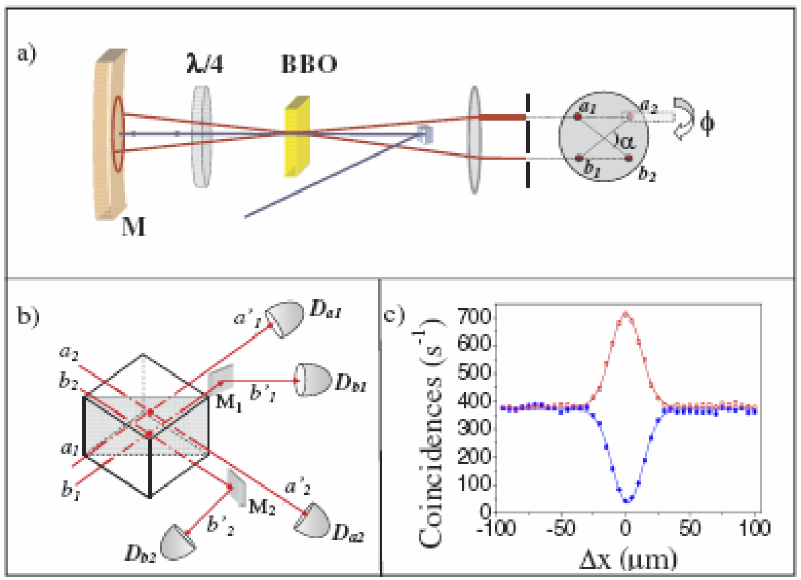
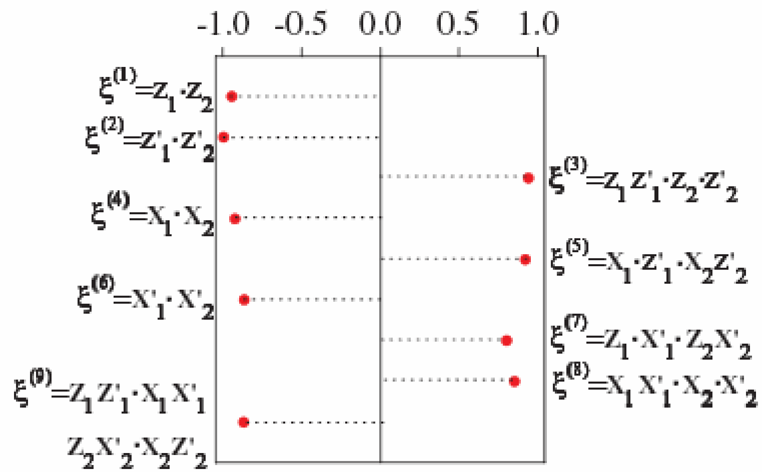




$$7 < \sum_{i=1}^{9}|\xi^{(i)}| = (8.114 \pm 0.011) < 9$$

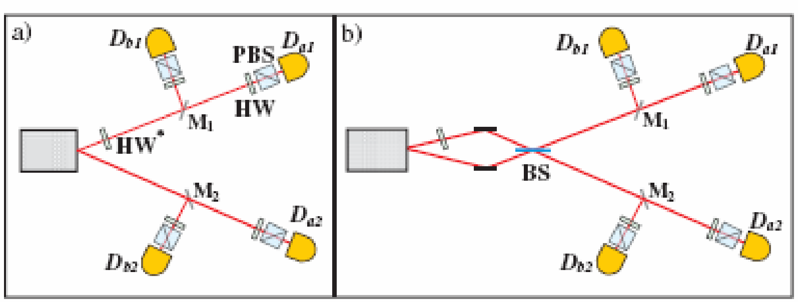
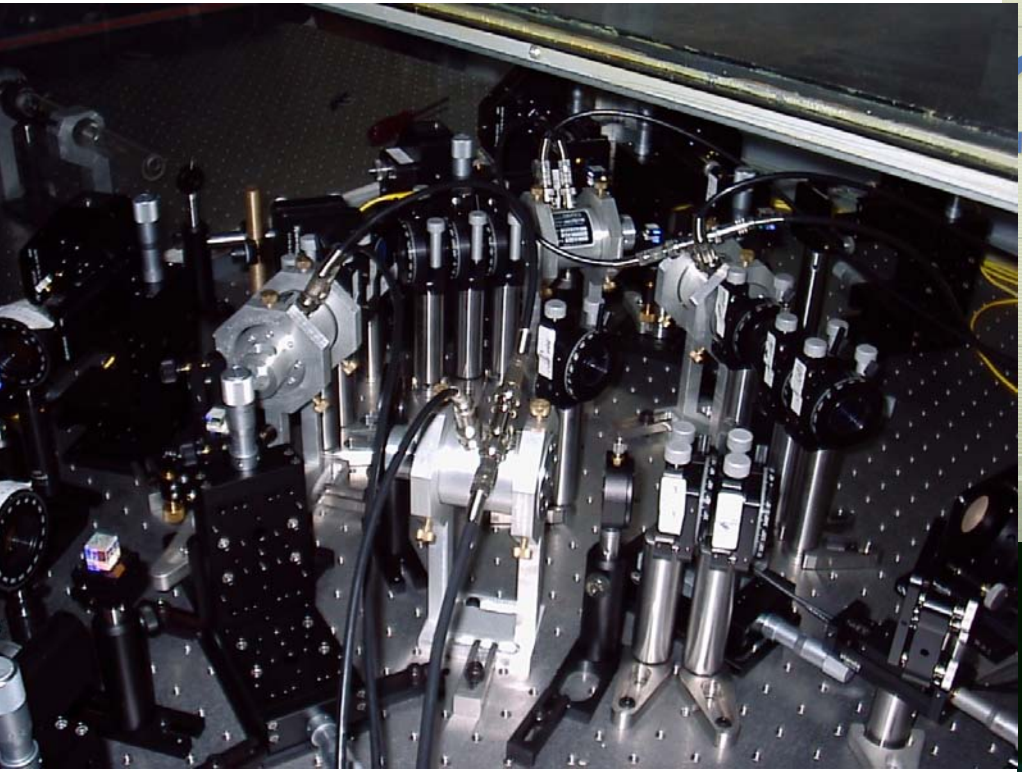


FIG. 3 (color online). Barchart of expectation values for the nine operators involved in the experiment. The following results have been obtained: $z_1 \cdot z_2 = -0.9428 \pm 0.0030$, $z'_1 \cdot z'_2 = -0.9953 \pm 0.0033$, $z_1 z'_1 \cdot z_2 \cdot z'_2 = 0.9424 \pm 0.0030$, $x_1 \cdot x_2 = -0.9215 \pm 0.0033$, $x_1 \cdot z'_1 \cdot x_2 z'_2 = 0.9217 \pm 0.0033$, $x'_1 \cdot x'_2 = -0.8642 \pm 0.0043$ $z_1 \cdot x'_1 \cdot z_2 x'_2 = 0.8039 \pm 0.0040$, $x_1 x'_1 \cdot x_2 \cdot x'_2 = 0.8542 \pm 0.0040$, $z_1 z'_1 \cdot x_1 x'_1 \cdot z_2 x'_2 \cdot x_2 z'_2 = -0.8678 \pm 0.0043$.

Hefei experiment 2005



Hefei experiment 2005

All-Versus-Nothing Violation of Local Realism by Two-Photon, Four-Dimensional Entanglement

Tao Yang,[1] Qiang Zhang,[1] Jun Zhang,[1] Juan Yin,[1] Zhi Zhao,[1,2] Marek Żukowski,[3]
Zeng-Bing Chen,[1,2,*] and Jian-Wei Pan[1,2,†]
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[3]*Instytut Fizyki Teoretycznej i Astrofizyki Uniwersytet Gdański, PL-80-952 Gdańsk, Poland*

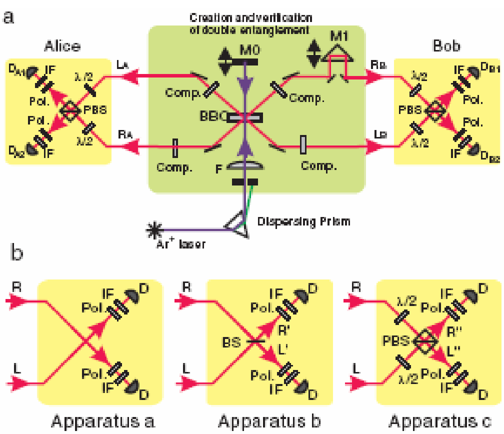


FIG. 1 (color online). Experimental setups. (a) An ultraviolet beam from Argon ion laser (351.1 nm, 120 mW) is directed into the BBO crystal from opposite directions, and thus can create photon pairs (with wavelength 702.2 nm) in $|\Psi\rangle$. Four compensators (Comp.) are used to offset the birefringent effect caused by the BBO crystal during parametric down-conversion. The reflection mirrors M0 and M1 are mounted on translation stages, to balance each arm of the interferometer and to optimize the entanglement in path. (b) Apparatuses to measure all necessary observables of doubly entangled states. D is single-photon count module, with collection and detection efficiency 26%; IF is interference filter with a bandwidth of 2.88 nm and a center wavelength of 702.2 nm; Pol. is polarizer. Apparatus c has been included in (a) at the locations of Alice and Bob.

$$z_A \cdot z_B|\Psi\rangle = -|\Psi\rangle, \qquad z'_A \cdot z'_B|\Psi\rangle = -|\Psi\rangle, \tag{1}$$

$$x_A \cdot x_B|\Psi\rangle = -|\Psi\rangle, \qquad x'_A \cdot x'_B|\Psi\rangle = -|\Psi\rangle, \tag{2}$$

$$z_A z'_A \cdot z_B \cdot z'_B|\Psi\rangle = |\Psi\rangle, \qquad x_A x'_A \cdot x_B \cdot x'_B|\Psi\rangle = |\Psi\rangle, \tag{3}$$

$$z_A \cdot x'_A \cdot z_B x'_B|\Psi\rangle = |\Psi\rangle, \qquad x_A \cdot z'_A \cdot x_B z'_B|\Psi\rangle = |\Psi\rangle, \tag{4}$$

$$z_A z'_A \cdot x_A x'_A \cdot z_B x'_B \cdot x_B z'_B|\Psi\rangle = -|\Psi\rangle. \tag{5}$$

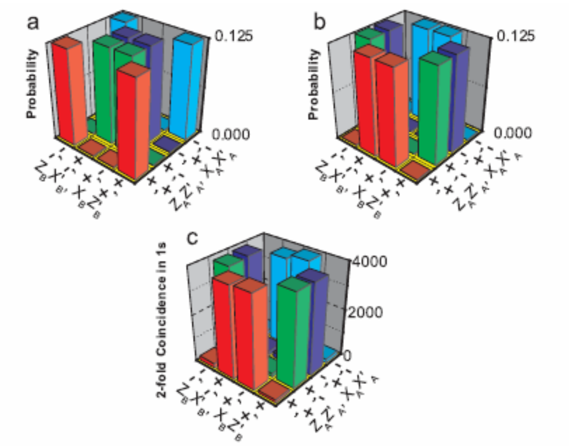


FIG. 3 (color online). Predictions of LR (a) and of QM (b), and observed results (c) for the $z_A z'_A \cdot x_A x'_A \cdot z_B x'_B \cdot x_B z'_B$ experiment.

Requires two-qubit measurements!

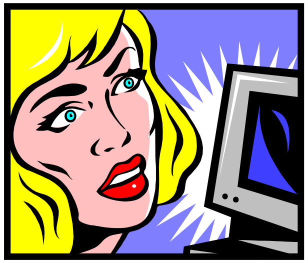
$$v(Z_1) = -v(Z_2)$$
$$v(z_1) = -v(z_2)$$
$$v(X_1) = -v(X_2)$$
$$v(x_1) = -v(x_2)$$
$$v(Z_1 z_1) = v(Z_2)v(z_2)$$
$$v(X_1 x_1) = v(X_2)v(x_2)$$
$$v(Z_1)v(x_1) = v(Z_2 x_2)$$
$$v(X_1)v(z_1) = v(X_2 z_2)$$
$$v(Z_1 z_1)v(X_1 x_1) = -v(Z_2 x_2)v(X_2 z_2)$$

Requires two-qubit measurements!

$$v(Z_1) = -v(Z_2)$$
$$v(z_1) = -v(z_2)$$
$$v(X_1) = -v(X_2)$$
$$v(x_1) = -v(x_2)$$
$$v(Z_1 z_1) = v(Z_2)v(z_2)$$
$$v(X_1 x_1) = v(X_2)v(x_2)$$
$$v(Z_1)v(x_1) = v(Z_2 x_2)$$
$$v(X_1)v(z_1) = v(X_2 z_2)$$
$$v(Z_1 z_1)v(X_1 x_1) = -v(Z_2 x_2)v(X_2 z_2)$$

Problem

- Two-observer AVN proof with single-qubit observables?



Two-observer AVN proof with single qubit observables

Hyperentangled cluster:

$$|HuHu\rangle + |HdHd\rangle + |VuVu\rangle - |VdVd\rangle$$

$$v(Z_1) = v(Z_2)$$
$$v(z_1) = v(z_2)$$
$$v(X_1) = v(X_2)v(z_2)$$
$$v(x_1) = v(Z_2)v(x_2)$$
$$v(Y_1) = -v(Y_2)v(z_2)$$
$$v(y_1) = -v(Z_2)v(y_2)$$
$$v(X_2) = v(X_1)v(z_1)$$
$$v(x_2) = v(Z_1)v(x_1)$$
$$v(Y_2) = -v(Y_1)v(z_1)$$
$$v(y_2) = -v(Z_1)v(y_1)$$

Two-observer AVN proof with single qubit observables

$$v(X_1) = v(X_2)\,v(z_2)$$

$$v(Y_1) = -\,v(Y_2)\,v(z_2)$$

$$v(X_1)\,v(x_1) = v(Y_2)\,v(y_2)$$

$$v(Y_1)\,v(x_1) = v(X_2)\,v(y_2)$$

Phys. Rev. Lett. **95**, 210401 (2005).

Rome experiment 2007

Realization and characterization of a 2-photon 4-qubit linear cluster state

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[2]Dipartimento Interateneo di Fisica, Università e Politecnico di Bari and Consorzio Nazionale Interuniversitario per le Scienze Fisiche della Materia, Bari, 70126 Italy

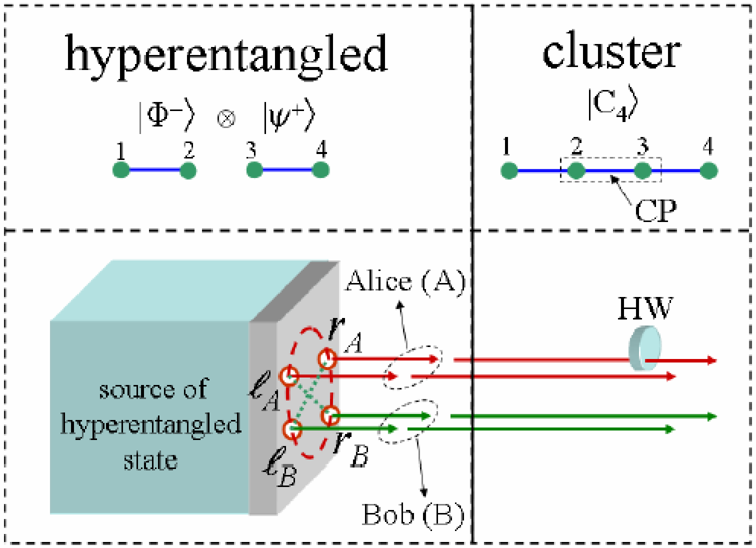


FIG. 1: Generation of the linear cluster state by a source of polarization-momentum hyperentangled 2-photon state. The state $|\Xi\rangle = |\Phi^-\rangle \otimes |\psi^+\rangle$ corresponds to two separate 2-qubit clusters. The *HW* acts as a Controlled-Phase (CP) thus generating the 4-qubit linear cluster $|C_4\rangle$.

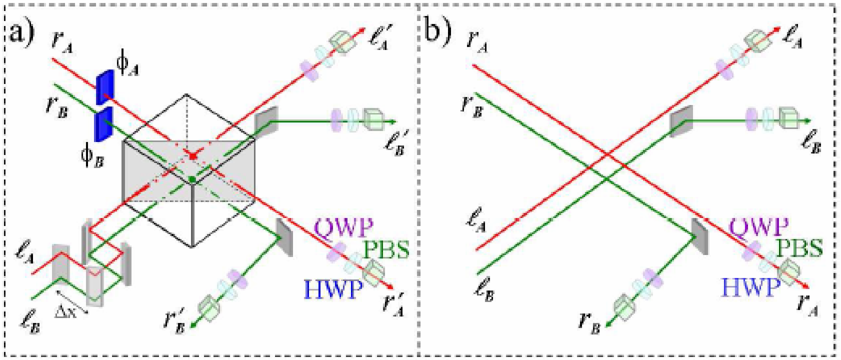


FIG. 2: Interferometer and measurement apparatus. a) The mode pairs r_A-ℓ_B and ℓ_A-r_B are matched on the BS. The phase shifters ϕ_A and ϕ_B (thin glass plates) are used for the measurement of momentum observables. The polarization analyzers on each of BS output modes are shown (QWP/HWP=Quarter/Half-Wave Plate, PBS=Polarized Beam Splitter). b) Same configuration as in a) with BS and glasses removed.

Phys. Rev. Lett. **98**, 180502 (2007).

Rome experiment 2007

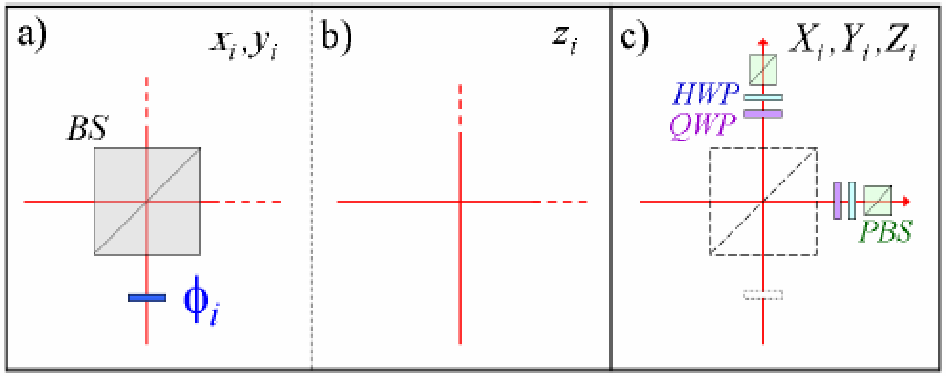


FIG. 4: Measurement setup for momentum (a),b)) and polarization (c)) observables for photon i (i=A, B). By the a) setup we measure x_i ($\phi_i = 0$) and y_i ($\phi_i = \frac{\pi}{2}$), while the b) setup is used for measuring z_i. By the c) setup we measure X_i ($\theta_Q = \frac{\pi}{4}$; $\theta_H = \frac{1}{8}\pi, \frac{3}{8}\pi$), Y_i ($\theta_Q = 0$; $\theta_H = \frac{1}{8}\pi, \frac{3}{8}\pi$) and Z_i ($\theta_Q = 0$; $\theta_H = 0, \frac{\pi}{4}$), where $\theta_{H(Q)}$ is the angle between the $HWP(QWP)$ optical axis and the vertical direction. The polarization analysis is performed contextually to x_i, y_i (i.e. with BS and glass) or z_i (without BS and glass), as shown by the dotted lines for BS and glass in c).

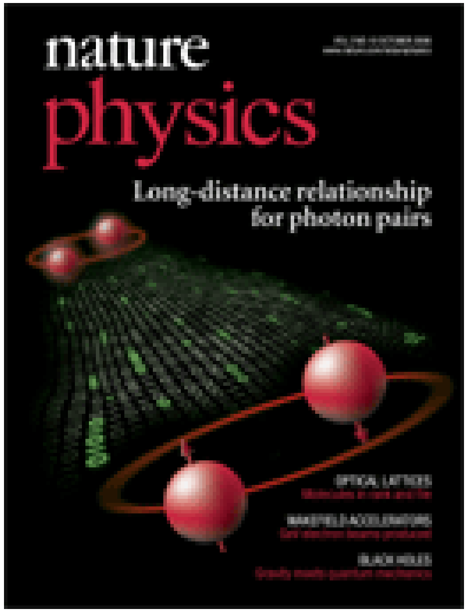
Observable	Value	$\mathcal{W}$	S	C
$Z_A Z_B$	$+0.9283 \pm 0.0032$	✓		
$Z_A x_A x_B$	$+0.8194 \pm 0.0049$	✓		
$X_A z_A X_B$	-0.9074 ± 0.0037	✓		✓
$z_A z_B$	-0.9951 ± 0.0009	✓		✓
$x_A Z_B x_B$	$+0.8110 \pm 0.0050$	✓		✓
$Z_A y_A y_B$	$+0.8071 \pm 0.0050$			✓
$Y_A z_A Y_B$	$+0.8948 \pm 0.0040$			✓
$X_A X_B z_B$	$+0.9074 \pm 0.0037$	✓	✓	✓
$Y_A Y_B z_B$	-0.8936 ± 0.0041		✓	✓
$X_A x_A Y_B y_B$	$+0.8177 \pm 0.0055$		✓	
$Y_A x_A X_B y_B$	$+0.7959 \pm 0.0055$		✓	

TABLE I: Experimental values of the observables used for measuring the entanglement witness $\mathcal{W}$ and the expectation value of S on the cluster state $|C_4\rangle$. The third column (C) refers to the control measurements needed to verify that X_A, Y_A, x_A, X_B, Y_B, y_B and z_B can be considered as elements of reality. Each experimental value corresponds to a measure lasting an average time of 10 sec. In the experimental errors we considered the poissonian statistic and the uncertainties due to the manual setting of the polarization analysis wave plates.

$$\mathrm{Tr}[S\rho_{exp}] = 3.4145 \pm 0.0095$$

Phys. Rev. Lett. **98**, 180502 (2007).

Motivation #1: Six-photon six-qubit states



• Q. Zhang, A. Goebel, C. Wagenknecht, Y.-A. Chen, B. Zhao, T. Yang, A. Mair, J. Schmiedmayer, and J.-W. Pan, "Experimental quantum teleportation of a two-qubit composite system",
Nature Physics **2**, 678 (2006).

• C.-Y. Lu, X.-Q. Zhou, O. Gühne, W.-B. Gao, J. Zhang, Z.-S. Yuan, A. Goebel, T. Yang, and J.-W. Pan,
"Experimental entanglement of six photons in graph states'",
Nature Physics **3**, 91 (2007).

• Other groups are preparing six-photon six-qubit states.

Motivation #2: Two-photon six-qubit states

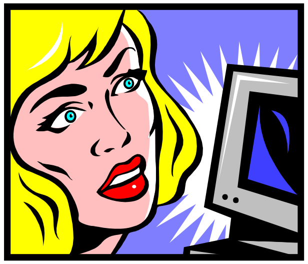
• J. T. Barreiro, N. K. Langford, N. A. Peters, and P. G. Kwiat,
“Hyper-entangled photons",
Phys. Rev. Lett. **95**, 260501 (2005).

$$\underbrace{(|HH\rangle + |VV\rangle)}_{\text{polarization}} \otimes \underbrace{(|rl\rangle + \alpha|gg\rangle + |lr\rangle)}_{\text{spatial modes}} \otimes \underbrace{(|ss\rangle + |ff\rangle)}_{\text{energy time}}.$$

• Other groups are preparing six-qubit two-photon hyper-entangled states.

Problem

• If we distribute *n* qubits between two parties,
what quantum pure <u>states and distributions</u> of qubits
allow AVN proofs using only single-qubit measurements?



First ingredient: Bipartite elements of reality

- Enough number of perfect correlations to define bipartite EPR's elements of reality. Every single-qubit observable involved in the proof must satisfy EPR's criterion; i.e., the result of measuring any of Alice's (Bob's) single-qubit observables must be possible to be predicted with certainty using the results of spacelike separated single-qubit measurements on Bob's (Alice's) qubits.

Example

Hyperentangled cluster:

$$|HuHu\rangle + |HdHd\rangle + |VuVu\rangle - |VdVd\rangle$$

$$v(Z_1) = v(Z_2)$$

$$v(z_1) = v(z_2)$$

$$v(X_1) = v(X_2)v(z_2)$$

$$v(x_1) = v(Z_2)v(x_2)$$

$$v(Y_1) = -v(Y_2)v(z_2)$$

$$v(y_1) = -v(Z_2)v(y_2)$$

$$v(X_2) = v(X_1)v(z_1)$$

$$v(x_2) = v(Z_1)v(x_1)$$

$$v(Y_2) = -v(Y_1)v(z_1)$$

$$v(y_2) = -v(Z_1)v(y_1)$$

Second ingredient: Algebraic contradiction

- Enough number of perfect correlations to reach into a contradiction with EPR's elements of reality. Any of the observables satisfying EPR's condition *cannot* have predefined results, because it is impossible to assign them values -1 or 1 satisfying all the perfect correlations predicted by QM.

Example

$$v(X_1) = v(X_2)\,v(z_2)$$

$$v(Y_1) = -\,v(Y_2)\,v(z_2)$$

$$v(X_1)\,v(x_1) = v(Y_2)\,v(y_2)$$

$$v(Y_1)\,v(x_1) = v(X_2)\,v(y_2)$$

Phys. Rev. Lett. **95**, 210401 (2005).

Perfect correlations, stabilizer and graph states

- Perfect correlations are needed to establish elements of reality and to prove that they are incompatible with QM.

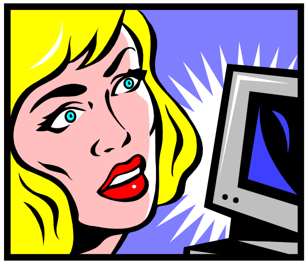
- Simultaneous eigenstates of a sufficiently large set of tensor operators products of single-qubit operators.

- We can restrict our attention to X, Y, Z. Stabilizer states!

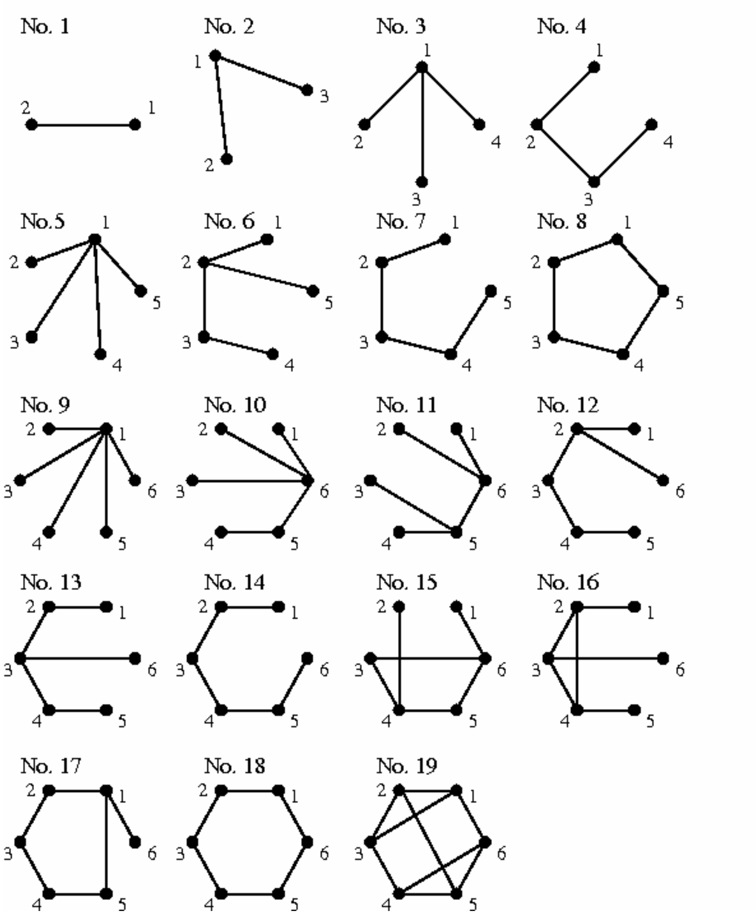
- Any stabilizer state is local Clifford equivalent to a graph state. Graph states!!

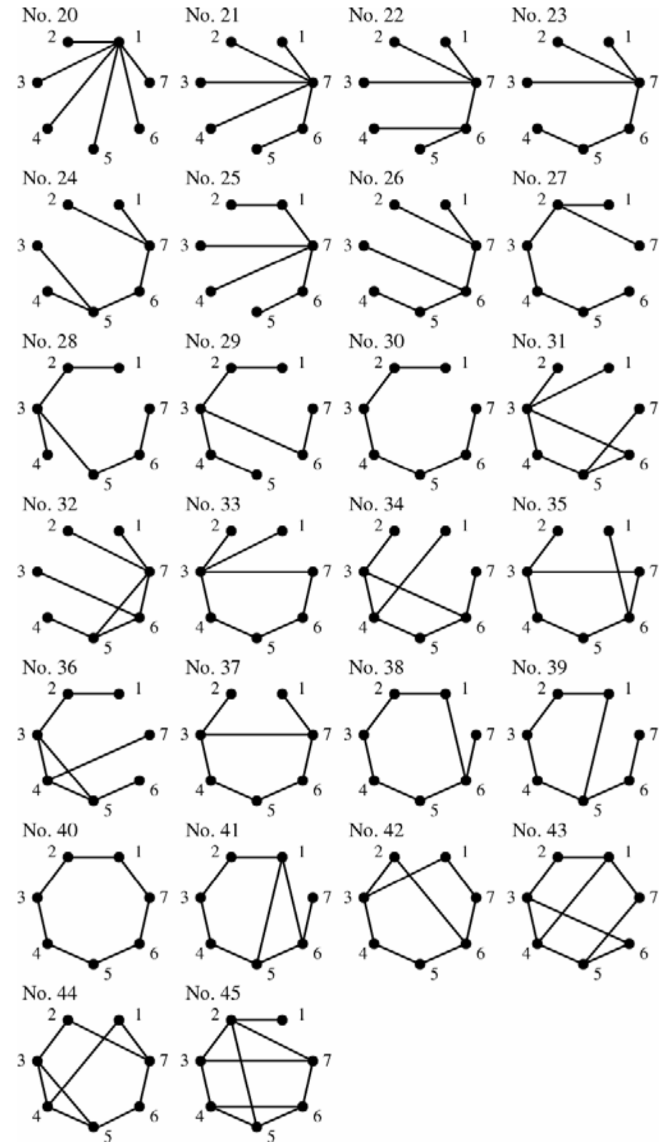
Problem

• If we distribute *n* qubits between two parties, what quantum graph <u>states and distributions</u> of qubits allow AVN proofs using only single-qubit measurements?



All graph states up to seven qubits





Graphs corresponding to all possible graph states, up to seven qubits, which are not equivalent under single-qubit transformations and graph isomorphism, M. Hein, J. Eisert, and H. J. Briegel, Phys. Rev. A **69**, 062311 (2004).

First ingredient: Bipartite elements of reality

Lemma: A distribution of n qubits between Alice (who is given n_A qubits) and Bob (who is given n_B qubits) permits bipartite elements of reality if and only if $n_A = n_B$, and the reduced stabilizer of Alice's (Bob's) qubits contains *all* possible variations with repetition of the four elements, $\mathbb{1}$, X, Y, and Z, choose n_A (n_B), and none of them repeated.

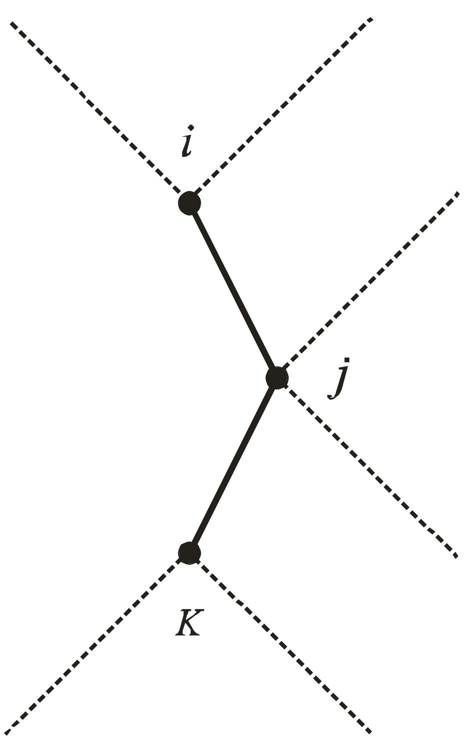
arXiv:0705.2613 [quant-ph].

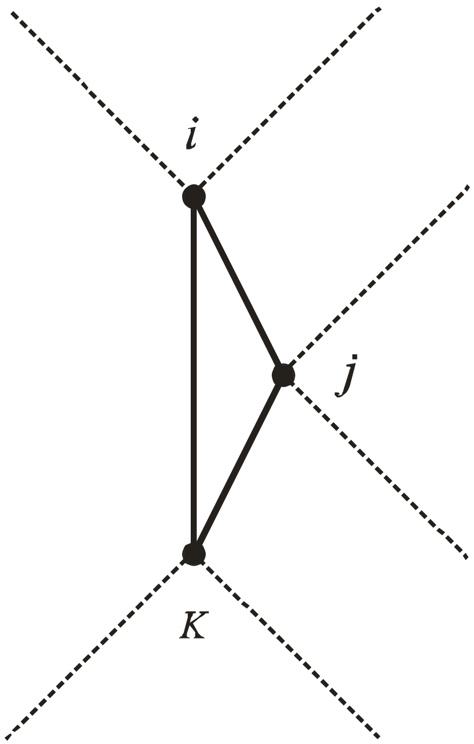
Second ingredient: Algebraic contradiction

Lemma: Any graph state associated to a connected graph of three or more vertices leads to an algebraic contradictions with the concept of elements of reality (when each qubit is distributed to a different party).

arXiv:0705.2613 [quant-ph].

Second ingredient: Examples of contradictions





$$
\begin{aligned}
g_i g_j |G\rangle &= |G\rangle \\
g_j |G\rangle &= |G\rangle \\
g_j g_k |G\rangle &= |G\rangle \\
g_i g_j g_k |G\rangle &= |G\rangle
\end{aligned}
$$

$$
\begin{aligned}
g_i |G\rangle &= |G\rangle \\
g_j |G\rangle &= |G\rangle \\
g_k |G\rangle &= |G\rangle \\
g_i g_j g_k |G\rangle &= |G\rangle
\end{aligned}
$$

Four-qubit cluster state

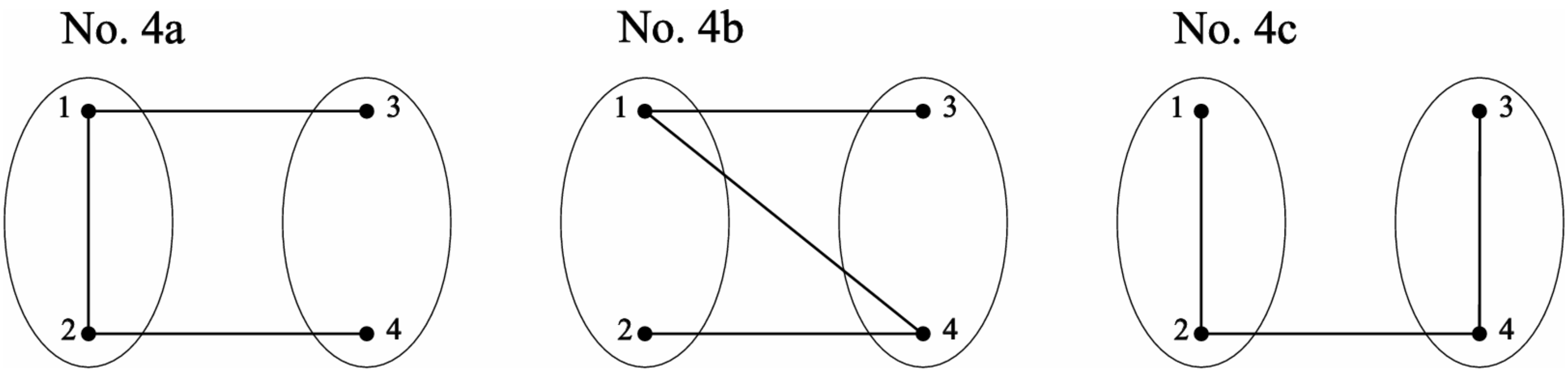
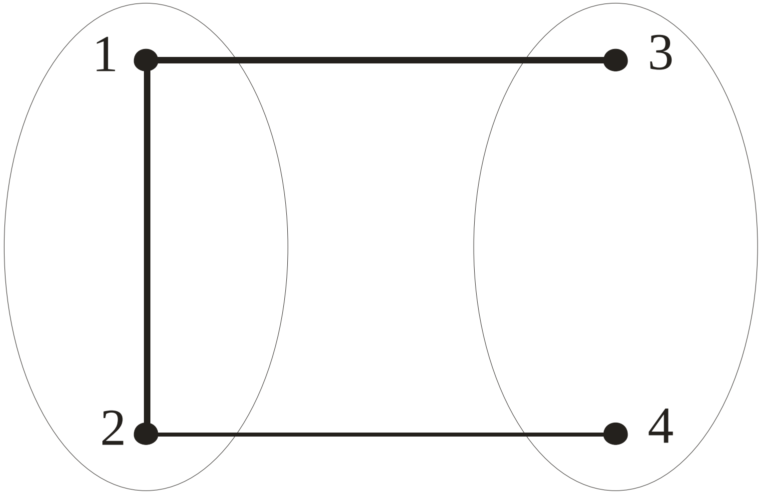


FIG. 1: Bipartite distributions of the 4-qubit linear cluster state (graph state no. 4 according to Hein *et al.* [28]). The distribution no. 4a permits bipartite elements of reality. The distribution 4b is physically equivalent (it is just relabeling the basis of the qubits). The distribution 4c is not equivalent to the other two, but does not permit bipartite elements of reality.

Four-qubit graph state allowing bipartite AVN proofs

No. 4a



$$|\psi_{4a}\rangle = \frac{1}{2}(|00\rangle|\bar{0}\bar{0}\rangle + |01\rangle|\bar{0}\bar{1}\rangle + |10\rangle|\bar{1}\bar{0}\rangle - |11\rangle|\bar{1}\bar{1}\rangle).$$

Alice's elements of reality	Bob's elements of reality
$X_1 = Z_3X_4$	$X_3 = Z_1$
$Y_1 = Y_3X_4$	$Y_3 = Y_1Z_2$
$Z_1 = X_3$	$Z_3 = X_1Z_2$
$X_2 = X_3Z_4$	$X_4 = Z_2$
$Y_2 = X_3Y_4$	$Y_4 = Z_1Y_2$
$Z_2 = X_4$	$Z_4 = Z_1X_2$

Bipartite AVN proof with single qubit observables…

$$v(X_1) = v(X_2)v(z_2)$$

$$v(Y_1) = -v(Y_2)v(z_2)$$

$$v(X_1)v(x_1) = v(Y_2)v(y_2)$$

$$v(Y_1)v(x_1) = v(X_2)v(y_2)$$

Phys. Rev. Lett. **95**, 210401 (2005).

… the only one with four qubits!!!!

$$v(X_1) = v(X_2)\,v(z_2)$$

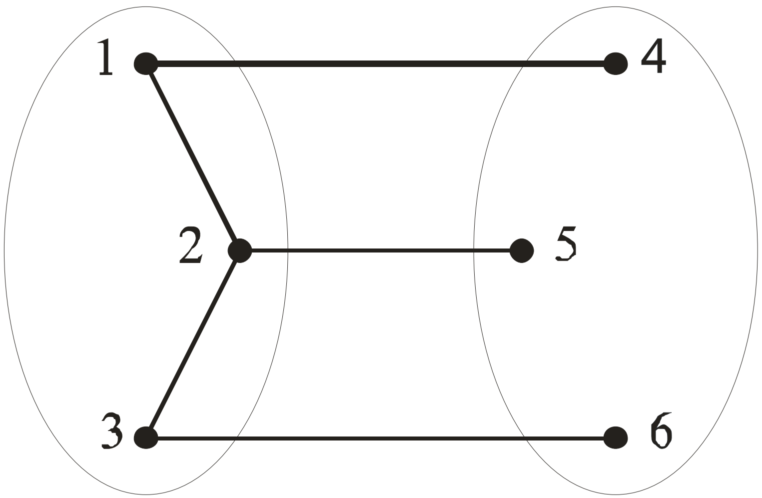
$$v(Y_1) = -\,v(Y_2)\,v(z_2)$$

$$v(X_1)\,v(x_1) = v(Y_2)\,v(y_2)$$

$$v(Y_1)\,v(x_1) = v(X_2)\,v(y_2)$$

Six-qubit graph states allowing bipartite AVN proofs

No. 13a

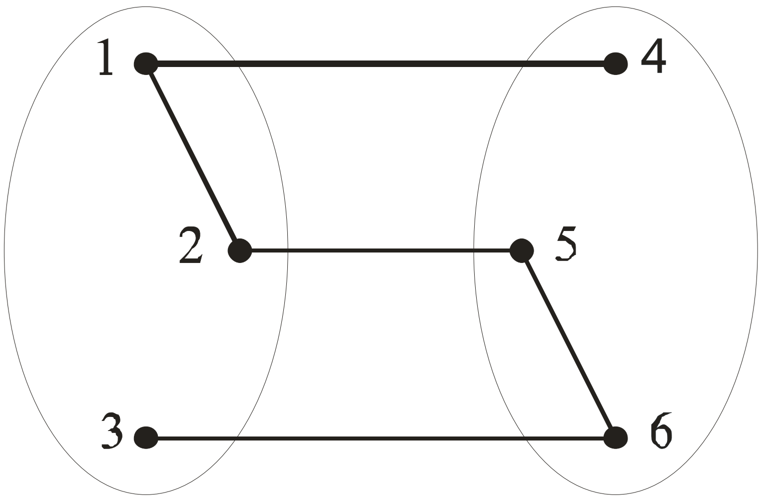


$$
\begin{aligned}
|\psi_{13a}\rangle \;=\; & \frac{1}{2\sqrt{2}}(|0\bar{0}0\bar{0}0\bar{0}\rangle + |0\bar{0}1\bar{0}1\bar{1}\rangle + |0\bar{1}0\bar{0}1\bar{0}\rangle \\
& +|0\bar{1}1\bar{0}0\bar{1}\rangle + |1\bar{0}0\bar{1}1\bar{0}\rangle + |1\bar{0}1\bar{1}0\bar{1}\rangle \\
& +|1\bar{1}0\bar{1}0\bar{0}\rangle + |1\bar{1}1\bar{1}1\bar{1}\rangle).
\end{aligned}
$$

Alice's elements of reality	Bob's elements of reality
$X_1 = Z_4 X_5$	$X_4 = Z_1$
$Y_1 = Y_4 X_5$	$Y_4 = Y_1 Z_2$
$Z_1 = X_4$	$Z_4 = X_1 Z_2$
$X_2 = X_4 Z_5 X_6$	$X_5 = Z_2$
$Y_2 = X_4 Y_5 X_6$	$Y_5 = Z_1 Y_2 Z_3$
$Z_2 = X_5$	$Z_5 = Z_1 X_2 Z_3$
$X_3 = X_5 Z_6$	$X_6 = Z_3$
$Y_3 = X_5 Y_6$	$Y_6 = Z_2 Y_3$
$Z_3 = X_6$	$Z_6 = Z_2 X_3$

Six-qubit graph states allowing bipartite AVN proofs

No. 14a

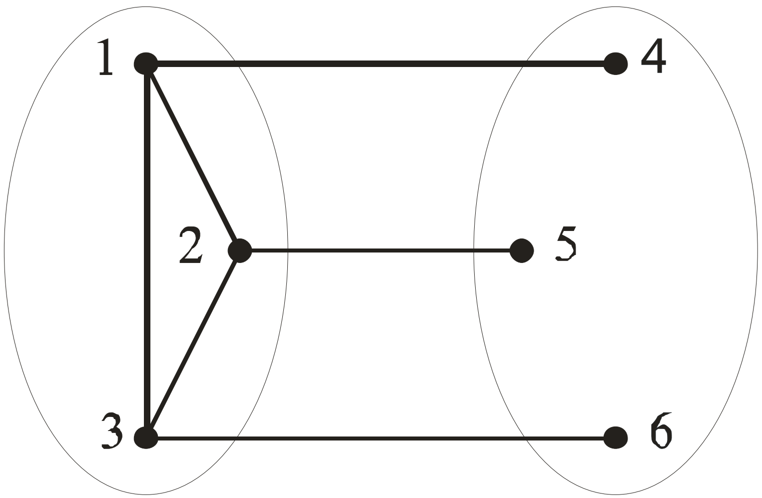


$$
\begin{aligned}
|\psi_{14a}\rangle \;=\; & \frac{1}{2\sqrt{2}}(|0\bar{0}0\bar{0}0\bar{0}\rangle + |0\bar{0}1\bar{0}0\bar{1}\rangle + |0\bar{1}0\bar{0}1\bar{1}\rangle \\
& +|0\bar{1}1\bar{0}1\bar{0}\rangle + |1\bar{0}0\bar{1}1\bar{1}\rangle + |1\bar{0}1\bar{1}1\bar{0}\rangle \\
& +|1\bar{1}0\bar{1}0\bar{0}\rangle + |1\bar{1}1\bar{1}0\bar{1}\rangle).
\end{aligned}
$$

Alice′s elements of reality	Bob′s elements of reality
$X_1 = Z_4 X_5 Z_6$	$X_4 = Z_1$
$Y_1 = Y_4 X_5 Z_6$	$Y_4 = Y_1 Z_2$
$Z_1 = X_4$	$Z_4 = X_1 Z_2$
$X_2 = X_4 Z_5$	$X_5 = Z_2 X_3$
$Y_2 = X_4 Y_5 Z_6$	$Y_5 = Z_1 Y_2 X_3$
$Z_2 = X_5 Z_6$	$Z_5 = Z_1 X_2$
$X_3 = Z_6$	$X_6 = Z_1 X_2 Z_3$
$Y_3 = Z_5 Y_6$	$Y_6 = Z_1 X_2 Y_3$
$Z_3 = X_6$	$Z_6 = X_3$

Six-qubit graph states allowing bipartite AVN proofs

No. 16a

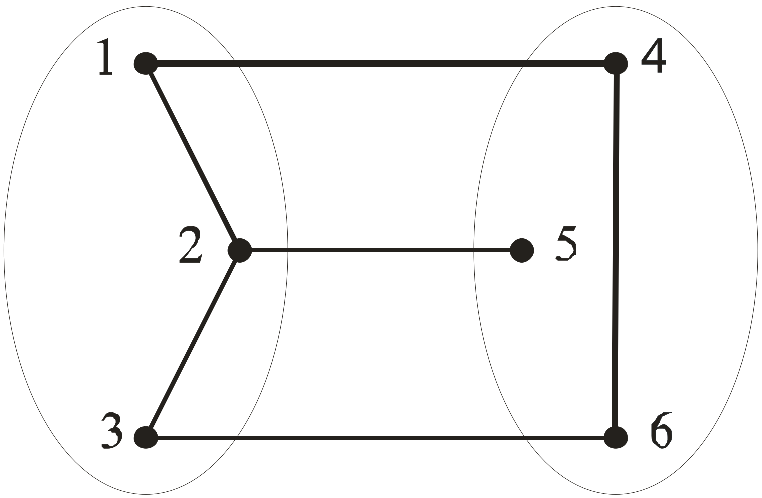


$$
\begin{aligned}
|\psi_{16a}\rangle \;=\; & \frac{1}{2\sqrt{2}}(|0\bar{0}0\bar{0}0\bar{0}\rangle + |0\bar{0}1\bar{0}1\bar{1}\rangle + |0\bar{1}0\bar{0}1\bar{0}\rangle \\
& +|0\bar{1}1\bar{0}0\bar{1}\rangle + |1\bar{0}0\bar{1}1\bar{0}\rangle - |1\bar{0}1\bar{1}0\bar{1}\rangle \\
& +|1\bar{1}0\bar{1}0\bar{0}\rangle - |1\bar{1}1\bar{1}1\bar{1}\rangle).
\end{aligned}
$$

Alice's elements of reality	Bob's elements of reality
$X_1 = Z_4X_5X_6$	$X_4 = Z_1$
$Y_1 = Y_4X_5X_6$	$Y_4 = Y_1Z_2Z_3$
$Z_1 = X_4$	$Z_4 = X_1Z_2Z_3$
$X_2 = X_4Z_5X_6$	$X_5 = Z_2$
$Y_2 = X_4Y_5X_6$	$Y_5 = Z_1Y_2Z_3$
$Z_2 = X_5$	$Z_5 = Z_1X_2Z_3$
$X_3 = X_4X_5Z_6$	$X_6 = Z_3$
$Y_3 = X_4X_5Y_6$	$Y_6 = Z_1Z_2Y_3$
$Z_3 = X_6$	$Z_6 = Z_1Z_2X_3$

Six-qubit graph states allowing bipartite AVN proofs

No. 17a

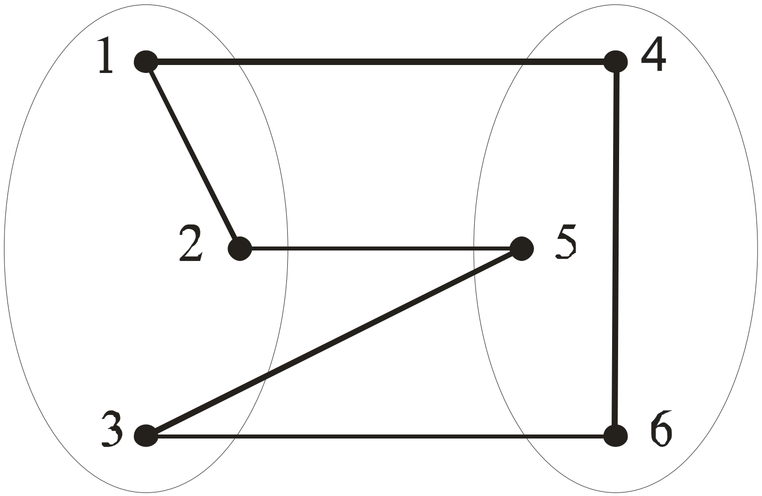


$$
\begin{aligned}
|\psi_{17a}\rangle \; = \; & \frac{1}{2\sqrt{2}}(|00\bar{0}\bar{0}\bar{0}0\rangle + |00\bar{1}\bar{1}\bar{0}1\rangle + |01\bar{0}\bar{1}\bar{1}1\rangle \\
& +|01\bar{1}\bar{0}\bar{1}0\rangle + |10\bar{0}\bar{1}\bar{0}0\rangle + |10\bar{1}\bar{0}\bar{0}1\rangle \\
& -|11\bar{0}\bar{0}\bar{1}1\rangle - |11\bar{1}\bar{1}\bar{1}1\rangle).
\end{aligned}
$$

Alice′s elements of reality	Bob′s elements of reality
$X_1 = Z_4 X_5$	$X_4 = Z_1 Z_2 X_3$
$Y_1 = Y_4 X_5 Z_6$	$Y_4 = Y_1 X_3$
$Z_1 = X_4 Z_6$	$Z_4 = X_1 Z_2$
$X_2 = Y_4 Z_5 Y_6$	$X_5 = Z_2$
$Y_2 = Y_4 Y_5 Y_6$	$Y_5 = Z_1 Y_2 Z_3$
$Z_2 = X_5$	$Z_5 = Z_1 X_2 Z_3$
$X_3 = X_5 Z_6$	$X_6 = X_1 Z_2 Z_3$
$Y_3 = Z_4 X_5 Y_6$	$Y_6 = X_1 Y_3$
$Z_3 = Z_4 X_6$	$Z_6 = Z_2 X_3$

Six-qubit graph states allowing bipartite AVN proofs

No. 18a

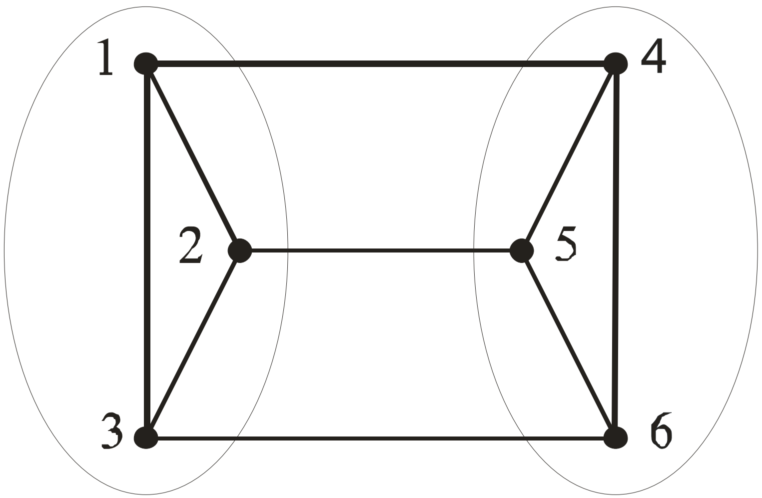


$$|\psi_{18a}\rangle = \frac{1}{2\sqrt{2}}(|\bar{0}00\bar{0}0\bar{0}\rangle + |\bar{0}01\bar{0}1\bar{1}\rangle + |\bar{0}10\bar{1}1\bar{1}\rangle + |\bar{0}11\bar{1}0\bar{0}\rangle + |\bar{1}00\bar{1}0\bar{1}\rangle + |\bar{1}01\bar{1}1\bar{0}\rangle + |\bar{1}10\bar{0}1\bar{0}\rangle + |\bar{1}11\bar{0}0\bar{1}\rangle).$$

Alice's elements of reality	Bob's elements of reality
$X_1 = X_5X_6$	$X_4 = X_2X_3$
$Y_1 = -X_4X_5Y_6$	$Y_4 = -X_1Y_2X_3$
$Z_1 = X_4Z_6$	$Z_4 = X_1Z_2$
$X_2 = X_4Z_5Z_6$	$X_5 = Z_2Z_3$
$Y_2 = Y_4Y_5Y_6$	$Y_5 = Z_1Y_2Z_3$
$Z_2 = Z_4X_5X_6$	$Z_5 = Z_1X_2$
$X_3 = Z_5Z_6$	$X_6 = X_1Z_2Z_3$
$Y_3 = Z_4Z_5Y_6$	$Y_6 = Y_1Y_2Y_3$
$Z_3 = Z_4X_6$	$Z_6 = Z_1X_2X_3$

Six-qubit graph states allowing bipartite AVN proofs

No. 19a

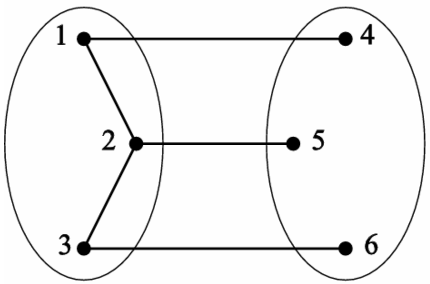


$$
\begin{aligned}
|\psi_{19a}\rangle = \frac{1}{4}(&|\bar{0}0000\bar{0}\rangle + |\bar{0}0001\bar{1}\rangle + |\bar{0}0110\bar{0}\rangle \\
&-|\bar{0}0111\bar{1}\rangle + |\bar{0}1010\bar{1}\rangle + |\bar{0}1011\bar{0}\rangle \\
&-|\bar{0}1100\bar{1}\rangle + |\bar{0}1101\bar{0}\rangle + |\bar{1}0010\bar{1}\rangle \\
&-|\bar{1}0011\bar{0}\rangle + |\bar{1}0100\bar{1}\rangle + |\bar{1}0101\bar{0}\rangle \\
&+|\bar{1}1000\bar{0}\rangle - |\bar{1}1001\bar{1}\rangle - |\bar{1}1110\bar{0}\rangle \\
&-|\bar{1}1111\bar{1}\rangle).
\end{aligned}
$$

Alice′s elements of reality	Bob′s elements of reality
$X_1 = Z_4 Y_5 Y_6$	$X_4 = Z_1 Y_2 Y_3$
$Y_1 = -Y_4 X_5 X_6$	$Y_4 = -Y_1 X_2 X_3$
$Z_1 = X_4 Z_5 Z_6$	$Z_4 = X_1 Z_2 Z_3$
$X_2 = Y_4 Z_5 Y_6$	$X_5 = Y_1 Z_2 Y_3$
$Y_2 = -X_4 Y_5 X_6$	$Y_5 = -X_1 Y_2 X_3$
$Z_2 = Z_4 X_5 Z_6$	$Z_5 = Z_1 X_2 Z_3$
$X_3 = Y_4 Y_5 Z_6$	$X_6 = Y_1 Y_2 Z_3$
$Y_3 = -X_4 X_5 Y_6$	$Y_6 = -X_1 X_2 Y_3$
$Z_3 = Z_4 Z_5 X_6$	$Z_6 = Z_1 Z_2 X_3$

Six-qubit graph states allowing bipartite AVN proofs

No. 13a



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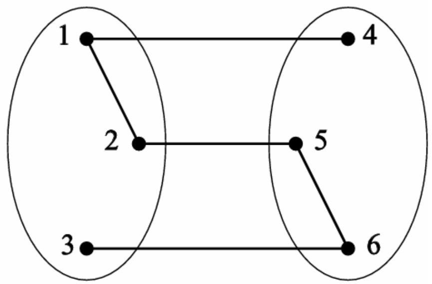

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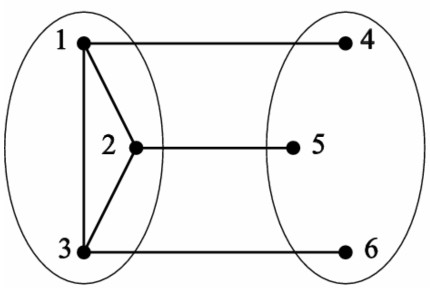

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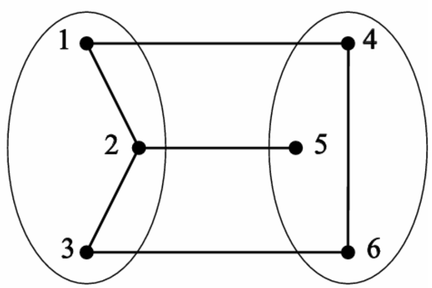

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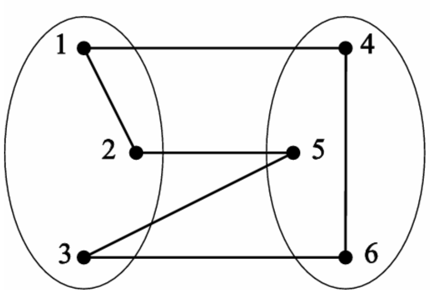

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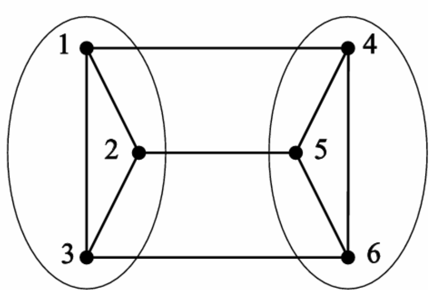


$$
\begin{aligned}
|\psi_{13a}\rangle &= \frac{1}{2\sqrt{2}}\big(|0\bar{0}0\bar{0}0\bar{0}\rangle + |0\bar{0}1\bar{0}1\bar{1}\rangle + |0\bar{1}0\bar{0}1\bar{0}\rangle \\
&\quad +|0\bar{1}1\bar{0}0\bar{1}\rangle + |1\bar{0}0\bar{1}1\bar{0}\rangle + |1\bar{0}1\bar{1}0\bar{1}\rangle \\
&\quad +|1\bar{1}0\bar{1}0\bar{0}\rangle + |1\bar{1}1\bar{1}1\bar{1}\rangle\big), \\
|\psi_{14a}\rangle &= \frac{1}{2\sqrt{2}}\big(|0\bar{0}0\bar{0}0\bar{0}\rangle + |0\bar{0}1\bar{0}0\bar{1}\rangle + |0\bar{1}0\bar{0}1\bar{1}\rangle \\
&\quad +|0\bar{1}1\bar{0}1\bar{0}\rangle + |1\bar{0}0\bar{1}1\bar{1}\rangle + |1\bar{0}1\bar{1}1\bar{0}\rangle \\
&\quad +|1\bar{1}0\bar{1}0\bar{0}\rangle + |1\bar{1}1\bar{1}0\bar{1}\rangle\big), \\
|\psi_{16a}\rangle &= \frac{1}{2\sqrt{2}}\big(|0\bar{0}0\bar{0}0\bar{0}\rangle + |0\bar{0}1\bar{0}1\bar{1}\rangle + |0\bar{1}0\bar{0}1\bar{0}\rangle \\
&\quad +|0\bar{1}1\bar{0}0\bar{1}\rangle + |1\bar{0}0\bar{1}1\bar{0}\rangle - |1\bar{0}1\bar{1}0\bar{1}\rangle \\
&\quad +|1\bar{1}0\bar{1}0\bar{0}\rangle - |1\bar{1}1\bar{1}1\bar{1}\rangle\big), \\
|\psi_{17a}\rangle &= \frac{1}{2\sqrt{2}}\big(|00\bar{0}\bar{0}\bar{0}0\rangle + |00\bar{1}\bar{1}\bar{0}1\rangle + |01\bar{0}\bar{1}\bar{1}1\rangle \\
&\quad +|01\bar{1}\bar{0}\bar{1}0\rangle + |10\bar{0}\bar{1}\bar{0}0\rangle + |10\bar{1}\bar{0}\bar{0}1\rangle \\
&\quad -|11\bar{0}\bar{0}\bar{1}1\rangle - |11\bar{1}\bar{1}\bar{1}1\rangle\big), \\
|\psi_{18a}\rangle &= \frac{1}{2\sqrt{2}}\big(|\bar{0}000\bar{0}\bar{0}\rangle + |\bar{0}010\bar{1}\bar{1}\rangle + |\bar{0}101\bar{1}\bar{1}\rangle \\
&\quad +|\bar{0}111\bar{0}\bar{0}\rangle + |\bar{1}001\bar{0}\bar{1}\rangle + |\bar{1}011\bar{1}\bar{0}\rangle \\
&\quad +|\bar{1}100\bar{1}\bar{0}\rangle + |\bar{1}110\bar{0}\bar{1}\rangle\big), \\
|\psi_{19a}\rangle &= \frac{1}{4}\big(|\bar{0}0000\bar{0}\rangle + |\bar{0}0001\bar{1}\rangle + |\bar{0}0110\bar{0}\rangle \\
&\quad -|\bar{0}0111\bar{1}\rangle + |\bar{0}1010\bar{1}\rangle + |\bar{0}1011\bar{0}\rangle \\
&\quad -|\bar{0}1100\bar{1}\rangle + |\bar{0}1101\bar{0}\rangle + |\bar{1}0010\bar{1}\rangle \\
&\quad -|\bar{1}0011\bar{0}\rangle + |\bar{1}0100\bar{1}\rangle + |\bar{1}0101\bar{0}\rangle \\
&\quad +|\bar{1}1000\bar{0}\rangle - |\bar{1}1001\bar{1}\rangle - |\bar{1}1110\bar{0}\rangle \\
&\quad -|\bar{1}1111\bar{1}\rangle\big).
\end{aligned}
$$

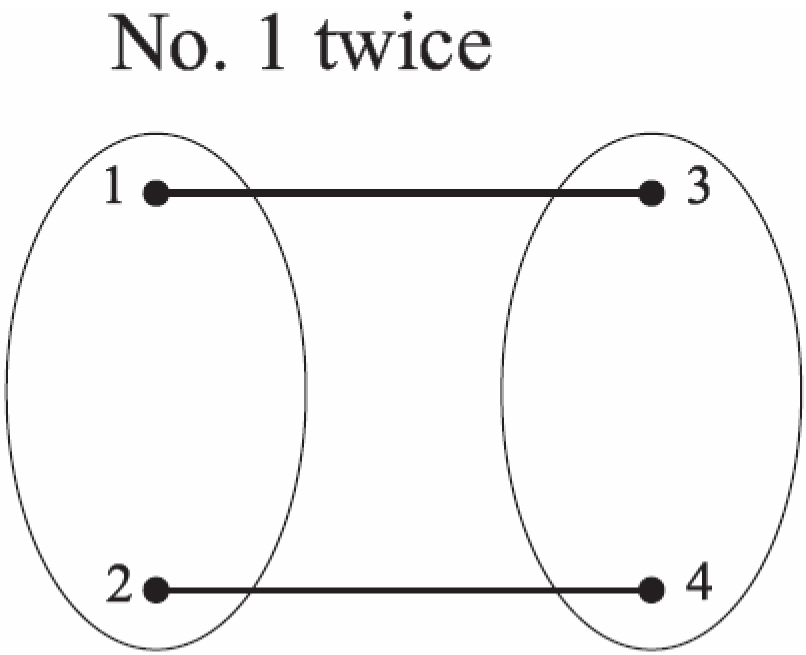
Problems

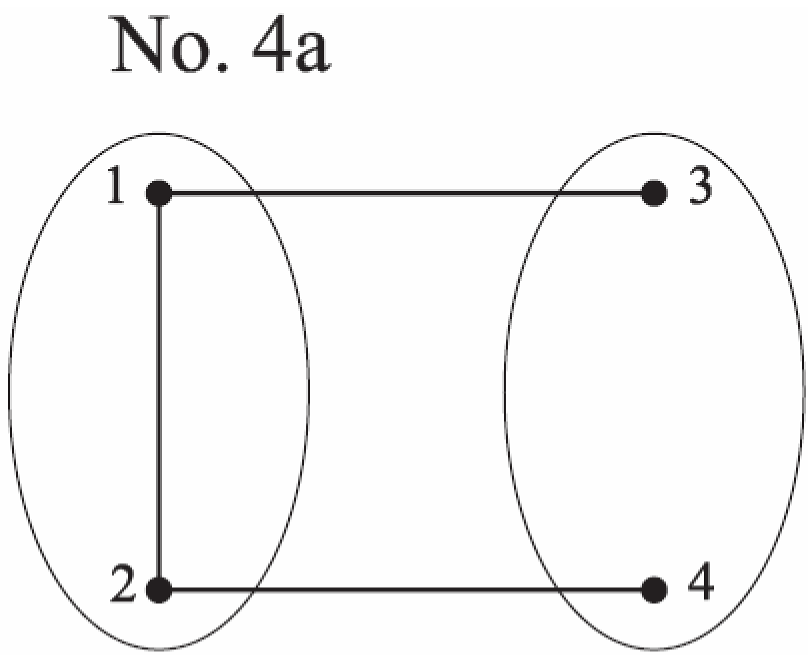
- Which is the maximum degree of nonlocality *D* for a six-qubit graph state allowing bipartite elements of reality?

- Which is the maximum degree of nonlocality *D* for the perfect correlations of a *n*-qubit graph state?

(*D* is defined as the ratio between the QM value and the bound of the Bell inequality. It is related to the minimum overall detection efficiency η required for a loophole-free experiment.)

Two-photon four-qubit experiments

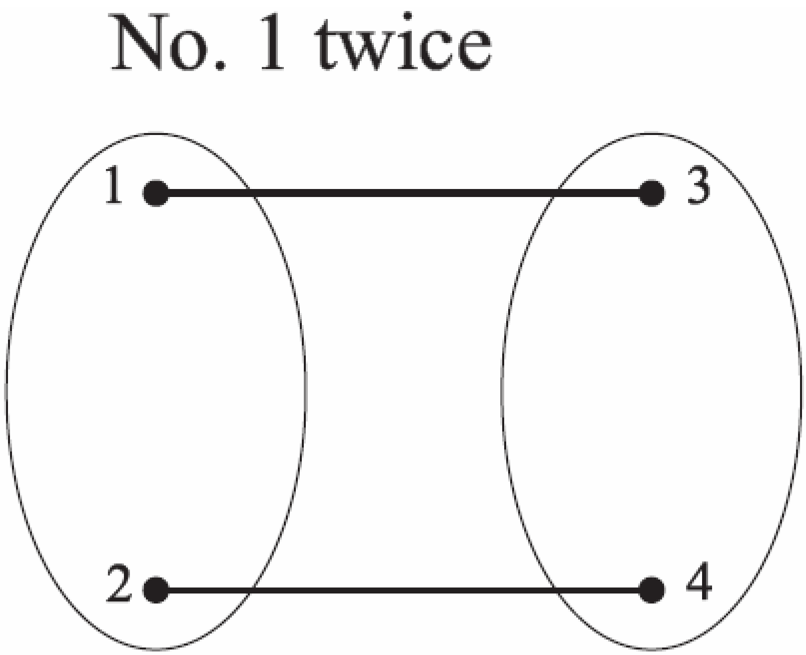




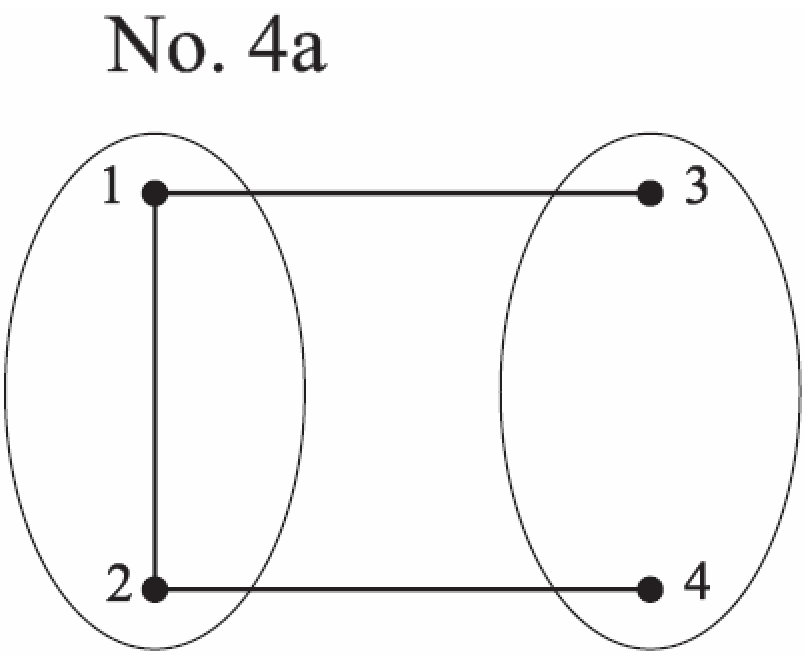
M. Barbieri, F. De Martini, P. Mataloni, G. Vallone, and A. Cabello, Phys. Rev. Lett. **97**, 140407 (2006).

G. Vallone, E. Pomarico, P. Mataloni, F. De Martini, and V. Berardi, Phys. Rev. Lett. **98**, 180502 (2007).

Two-photon four-qubit experiments

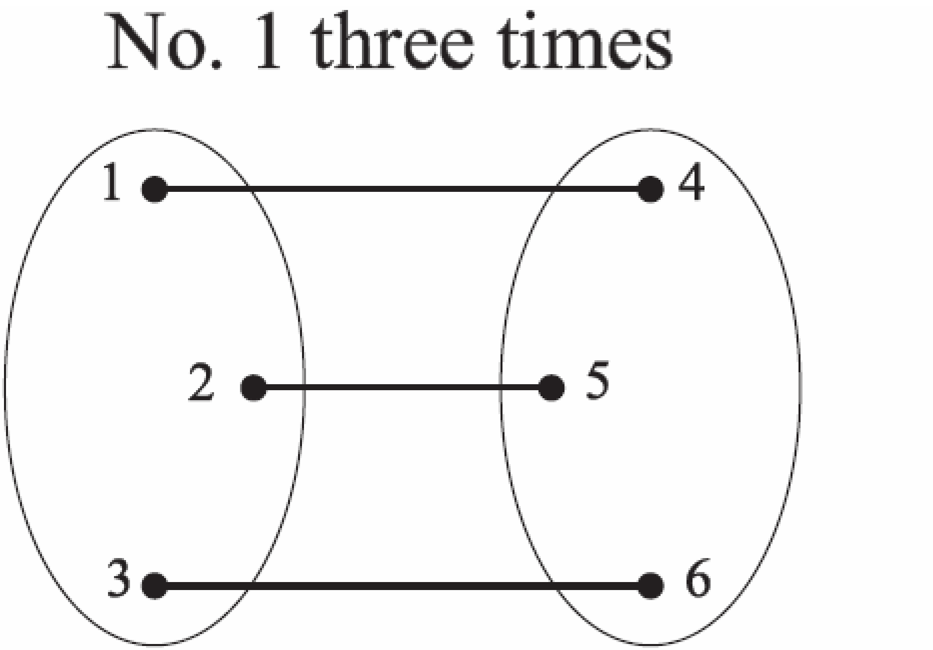


D = 2, 16 terms; η > 0.67



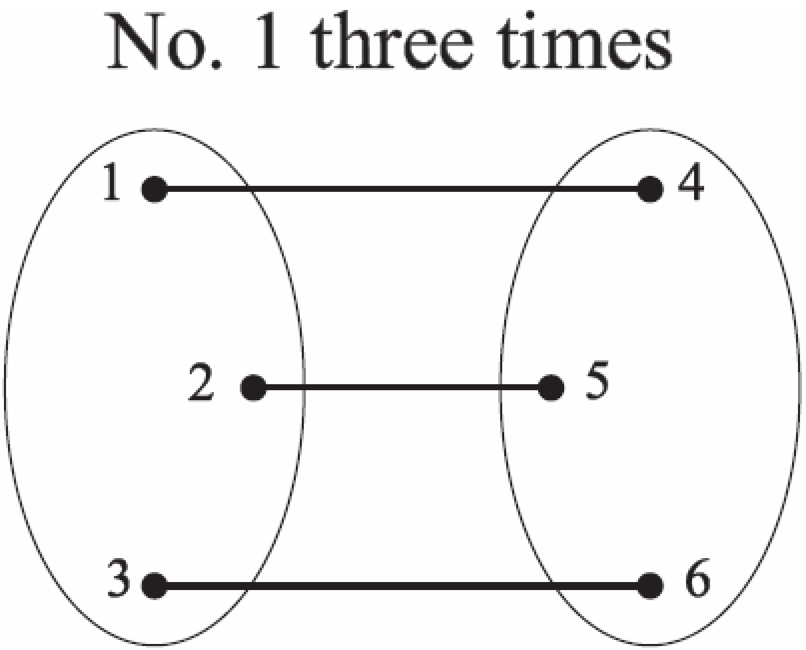
D = 2, 4 terms; η > 0.67

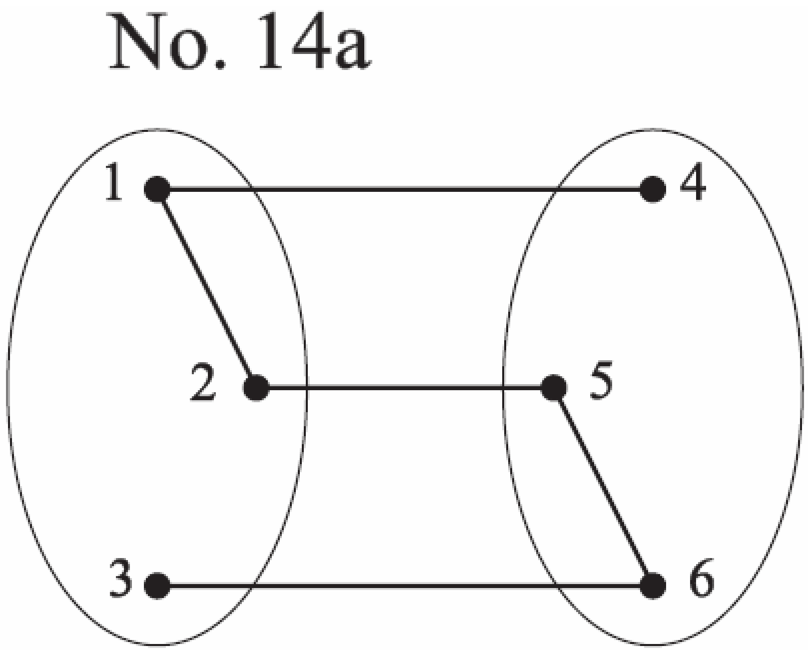
Two-photon six-qubit experiments



D = 2.8, 64 terms; η > 0.53

Two-photon six-qubit experiments

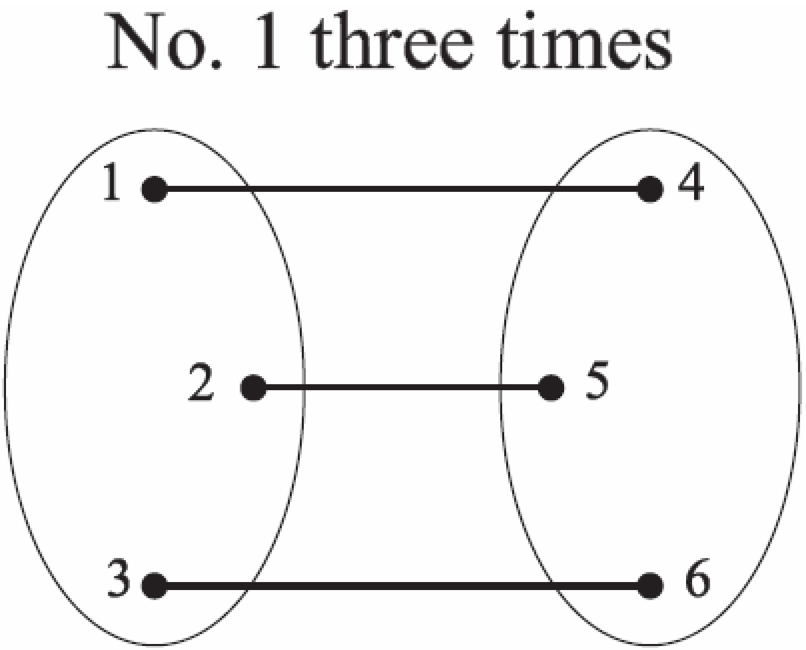




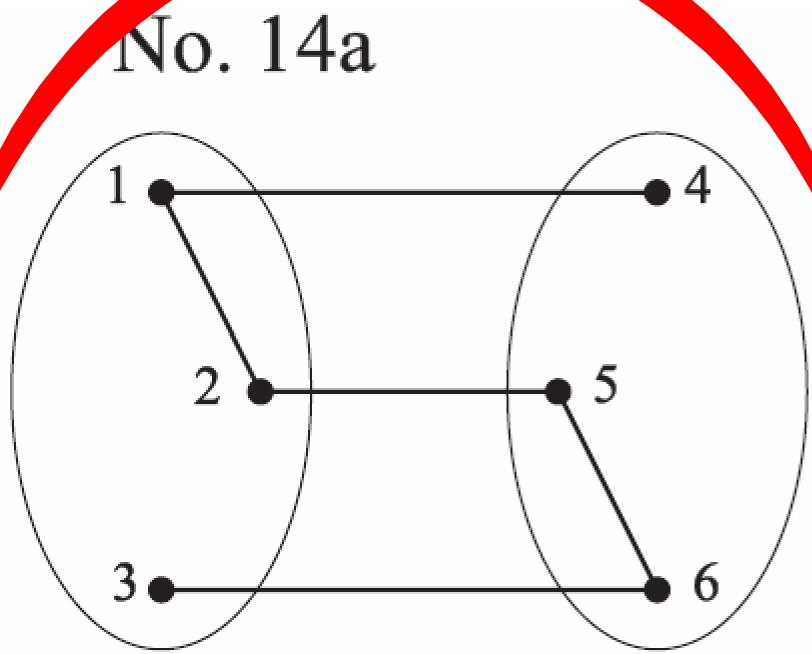
D = 2.8, 64 terms; η > 0.53

D = 4, 16 terms; η > 0.40

Two-photon six-qubit experiments



D = 2.8, 64 terms; η > 0.53



D = 4, 16 terms; η > 0.40

Plan

- Graph states
- EPR, Bell and the loophole-free experiment
- GHZ and exponentially growing with size nonlocality
- Bipartite AVN and the loophole-free experiment
- Optimal Bell inequalities for graph states

Motivation

PRL 95, 120405 (2005) PHYSICAL REVIEW LETTERS week ending 16 SEPTEMBER 2005

Bell Inequalities for Graph States

Otfried Gühne,[1] Géza Tóth,[2] Philipp Hyllus,[3,4] and Hans J. Briegel[1,5]

[1]*Institut für Quantenoptik und Quanteninformation, Österreichische Akademie der Wissenschaften, A-6020 Innsbruck, Austria*
[2]*Max-Planck-Institut für Quantenoptik, Hans-Kopfermann-Straße 1, D-85748 Garching, Germany*
[3]*Institut für Theoretische Physik, Universität Hannover, Appelstraße 2, D-30167 Hannover, Germany*
[4]*Institute for Mathematical Sciences, Imperial College London, 48 Prince's Gardens, SW7 London 2PE, United Kingdom*
[5]*Institut für Theoretische Physik, Universität Innsbruck, Technikerstraße 25, A-6020 Innsbruck, Austria*

(Received 13 October 2004; published 14 September 2005)

We investigate the nonlocal properties of graph states. To this aim, we derive a family of Bell inequalities which require three measurement settings for each party and are maximally violated by graph states. In turn, for each graph state there is an inequality maximally violated only by that state. We show that for certain types of graph states the violation of these inequalities increases exponentially with the number of qubits. We also discuss connections to other entanglement properties such as the positivity of the partial transpose or the geometric measure of entanglement.

$$
\begin{aligned}
\mathcal{B}(FC_3) = {} & \mathbb{1}^{(1)}\mathbb{1}^{(2)}\mathbb{1}^{(3)} + X^{(1)}Z^{(2)}Z^{(3)} + Z^{(1)}X^{(2)}Z^{(3)} \\
& + Z^{(1)}Z^{(2)}X^{(3)} + Y^{(1)}Y^{(2)}\mathbb{1}^{(3)} + Y^{(1)}\mathbb{1}^{(2)}Y^{(3)} \\
& + \mathbb{1}^{(1)}Y^{(2)}Y^{(3)} - X^{(1)}X^{(2)}X^{(3)}.
\end{aligned}
$$

Motivation

PRL 95, 120405 (2005) PHYSICAL REVIEW LETTERS week ending 16 SEPTEMBER 2005

Bell Inequalities for Graph States

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[2]*Max-Planck-Institut für Quantenoptik, Hans-Kopfermann-Straße 1, D-85748 Garching, Germany*

[3]*Institut für Theoretische Physik, Universität Hannover, Appelstraße 2, D-30167 Hannover, Germany*

[4]*Institute for Mathematical Sciences, Imperial College London, 48 Prince's Gardens, SW7 London 2PE, United Kingdom*

[5]*Institut für Theoretische Physik, Universität Innsbruck, Technikerstraße 25, A-6020 Innsbruck, Austria*

(Received 13 October 2004; published 14 September 2005)

We investigate the nonlocal properties of graph states. To this aim, we derive a family of Bell inequalities which require three measurement settings for each party and are maximally violated by graph states. In turn, for each graph state there is an inequality maximally violated only by that state. We show that for certain types of graph states the violation of these inequalities increases exponentially with the number of qubits. We also discuss connections to other entanglement properties such as the positivity of the partial transpose or the geometric measure of entanglement.

$$\mathcal{B}(FC_3) = X^{(1)}Z^{(2)}Z^{(3)} + Z^{(1)}X^{(2)}Z^{(3)} + Z^{(1)}Z^{(2)}X^{(3)} - X^{(1)}X^{(2)}X^{(3)}.$$

Optimal Bell inequalities for graph states

- Gühne et al.:

$$\beta = \sum_{j=1}^{2^N} s_j \qquad s_j|\psi_N\rangle = |\psi_N\rangle$$

- This work: Find the one with the largest degree of nonlocality of the family

$$\beta_k = \sum_{j=1}^{2^N-1} a_{kj} s_j, \qquad a_{kj} \in \{0, 1\}$$

Optimal Bell inequalities for graph states

TABLE I: All optimal Bell inequalities for the perfect correlations of all inequivalent graph states up to 5 qubits. $\mathcal{D}$ is the degree of nonlocality.

Graph no.	$\mathcal{D}$	Terms	Settings	(Symmetric) inequalities
2 (GHZ_3)	2	4	2-2-2	(1) 1
3 (GHZ_4)	2	4	2-2-2-2	(2) 8
4 (LC_4)	2	4	2-2-2-1	(0) 8
5 (GHZ_5)	4	16	2-2-2-2-2	(1) 1
6 (Y_5)	$15/7 \approx 2.14$	15	3-3-3-3-2 or 3-3-3-3-3	(2) 18 or (2) 18
7 (LC_5)	$5/2 = 2.5$	20	3-3-3-3-3	(1) 1
8 (RC_5)	$7/3 \approx 2.33$	21	3-3-3-3-3	(1) 6

arXiv:0708.3208 [quant-ph].

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- A. Cabello and P. Moreno, “Bipartite all-versus-nothing proofs of Bell's theorem with single-qubit measurements”, arXiv:0705.2613 [quant-ph].

- A. Cabello, O. Gühne, and D. Rodríguez, “Optimal Bell inequalities for perfect correlations”, 0708.3208 [quant-ph].

- A. Cabello and O. Gühne, “Extreme and robust exponentially-growing-with-size bi- and multipartite nonlocality”, work in progress.