Towards linear phononics, quantum information processing and nonlocality tests in ion traps

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CV quantum information

Quantum information with systems described by canonical operators, like $\hat{x}$ and $\hat{p}$, with continuous spectra
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‘Massive’ CV degrees of freedom

Controllable continuous variable degrees of freedom are ubiquitous
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Our pick are the “transverse” (or “radial”) degrees of freedom of trapped ions (Zhu, Monroe, Duan, PRL ’06; C. F. Roos et al., arXiv:0705.0788).

* courtesy of R. Blatt
“Linear optical” operations:

CV states can be efficiently:

- displaced (classical currents)
- squeezed ($\chi^2$ crystals)
- rotated (phase plates, beam splitters)
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How does one implement “linear optical” operations on trapped ions?
Linear phononics: the lonely ion

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The idea: control the trapping frequency
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The idea: control the trapping frequency

One ion:

\[ H_0 = \frac{p^2}{2m} + \frac{1}{2}m\omega_0^2 x^2 \]

\[ X \equiv \sqrt{m\omega_0}x \quad P \equiv \frac{p}{\sqrt{m\omega_0}} \Rightarrow H_0 = \frac{1}{2}\omega_0(P^2 + X^2) \]
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Any linear optical operation on a single particle can be implemented by controlling \( \omega \)
Linear phononics: the more the merrier

Any ‘linear optical’ operation can be implemented if individual control of the radial trapping frequencies is achieved (Serafozzi, Retzker, Plenio, arXiv:0708.0851)
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Non-resonant atoms: Coulomb interaction is suppressed
Linear phononics: the more the merrier

- **Any** ‘linear optical’ operation can be implemented if *individual* control of the radial trapping frequencies is achieved (Serafozzi, Retzker, Plenio, arXiv:0708.0851)
- At resonance: Coulomb interaction is on

![Diagram](image-url)
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- All the results about Gaussian states can be carried over to ion traps (harmonic approximation)
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- Possibility to go beyond Gaussian when anharmonicities kick in
Entanglement generation

Achieving local control of the frequencies could be challenging experimentally.
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- Achieving local control of the frequencies could be challenging experimentally.
- From now on, only *global* control is assumed: all the trapping frequencies are the same at all times (but can be changed all together).
Entanglement generation

Three ions: starting from the ground state for trapping frequency $\omega_i = 100 \text{ MHz}$ and evolving with frequency $\omega_f = 2 \text{ MHz}$, $T \simeq 21^\circ C$. (LogNeg between ion 1 and 3)
Entanglement generation

- **Two ions**: starting from the ground state for trapping frequency $\omega_i = 100 \text{ MHz}$ and evolving with frequency $\omega_f = 2 \text{ MHz}$, $T \simeq 21^\circ C$. 

![Graph showing time evolution of logarithmic negativity with different gamma values](image-url)
Entanglement generation

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- If swapped to light: entanglement generator for quantum optics
Nonlocality test

- Measurements can also be implemented: parity measurable in single runs
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⇒ the violation of Bell inequalities can be tested with Gaussian states

- Violation of ‘Bell-Klyshko’ inequality by displaced parity (Banaszek and Wodkiewicz, PRA ’98), for 3 ions:
Nonlocality test

- Measurements can also be implemented: parity measurable in single runs
- \( \Rightarrow \) the violation of Bell inequalities can be tested with Gaussian states
- Violation of ‘Bell-Klyshko’ inequality by displaced parity (Banaszek and Wodkiewicz, PRA ’98), for 3 ions:

\[
\begin{array}{ccccccc}
& -6 & -5 & -4 & -3 & -2 & -1 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\
B_3 & 2.5 & 2.4 & 2.3 & 2.2 & 2.1 & 2.0 & & & & & & & & \\
x_1(\text{nm}) & -6 & -5 & -4 & -3 & -2 & -1 & 0 & 1 & 2 & 3 & -3 & -2 & -1 & 0 & \\
x_2(\text{nm})
\end{array}
\]

effect of thermal noise:
\[ T \simeq 21^\circ C, \gamma N \simeq 200\text{Hz} \]
Summing up:

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⇒ Promising both for quantum information processing and as probes of fundamental physics