Lost in Translation

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Overview

- *Two models for Quantum Computing:*

➡ Quantum Circuits

➡ Measurement-based quantum computing (MBQC)

- *Structural Relations*

➡ Causal Structure

➡ Parallelism

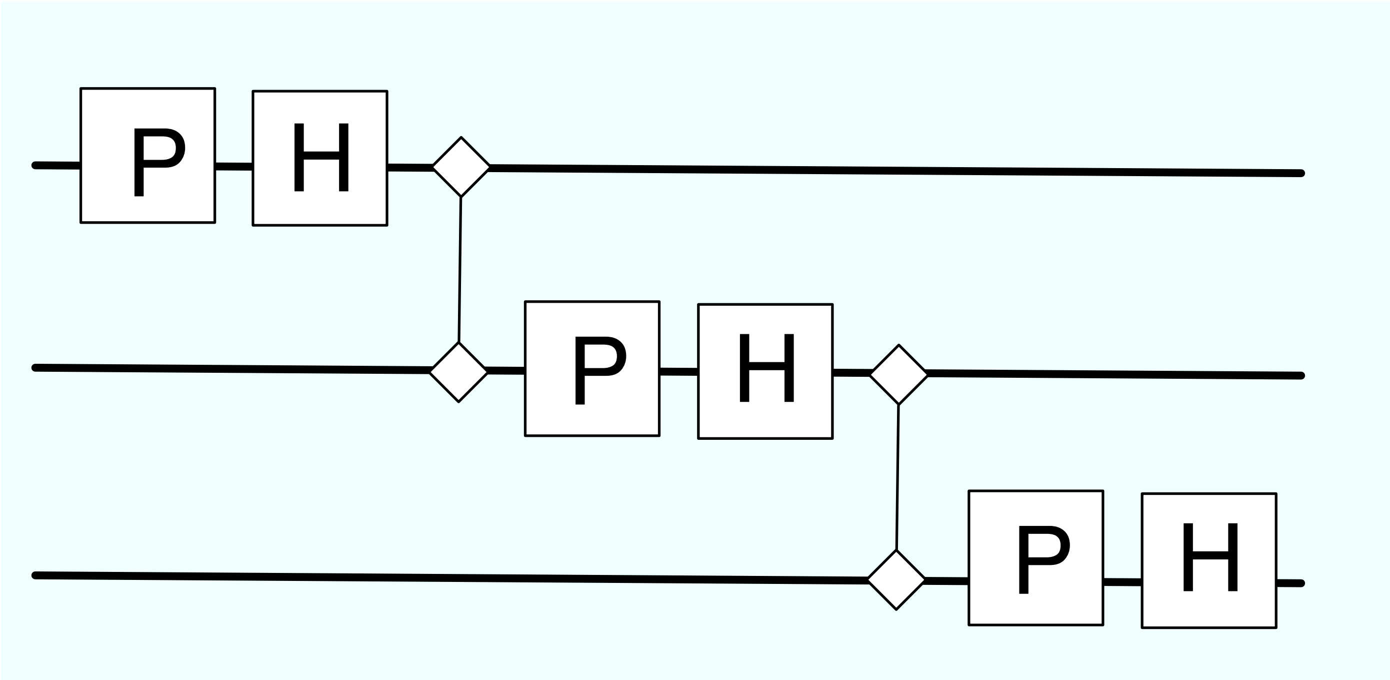
Quantum Circuit

A directed **acyclic** graph where degree 1 nodes are either input or output and other nodes are unitary gate. An arbitrary subset of the inputs (outputs) are labelled *auxiliary (result*).

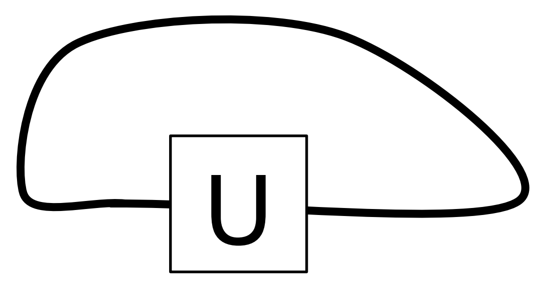
$$H := \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

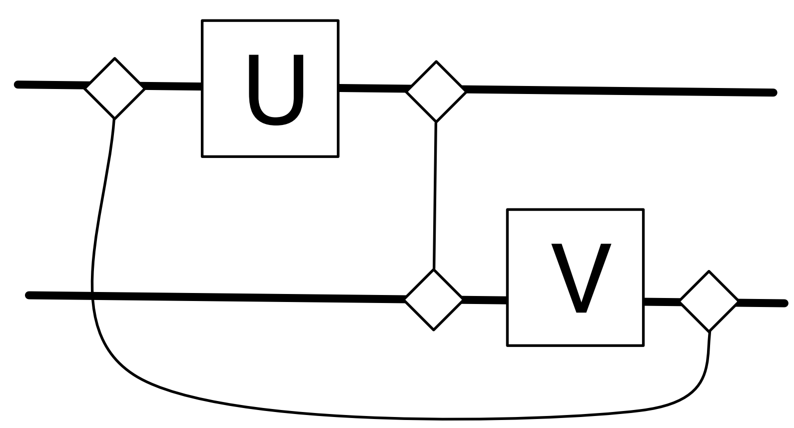
$$P(\alpha) := \begin{pmatrix} 1 & 0 \\ 0 & e^{i\alpha} \end{pmatrix}$$

$$\wedge Z = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$



Cyclic Quantum Circuit





Quantum Circuit with Measurements

Theorem. [Aharonov, Kitaev, Nisan] Qcircuit with measurement gates is computationally equivalent to Qcircuit with measurements performed only at the end.

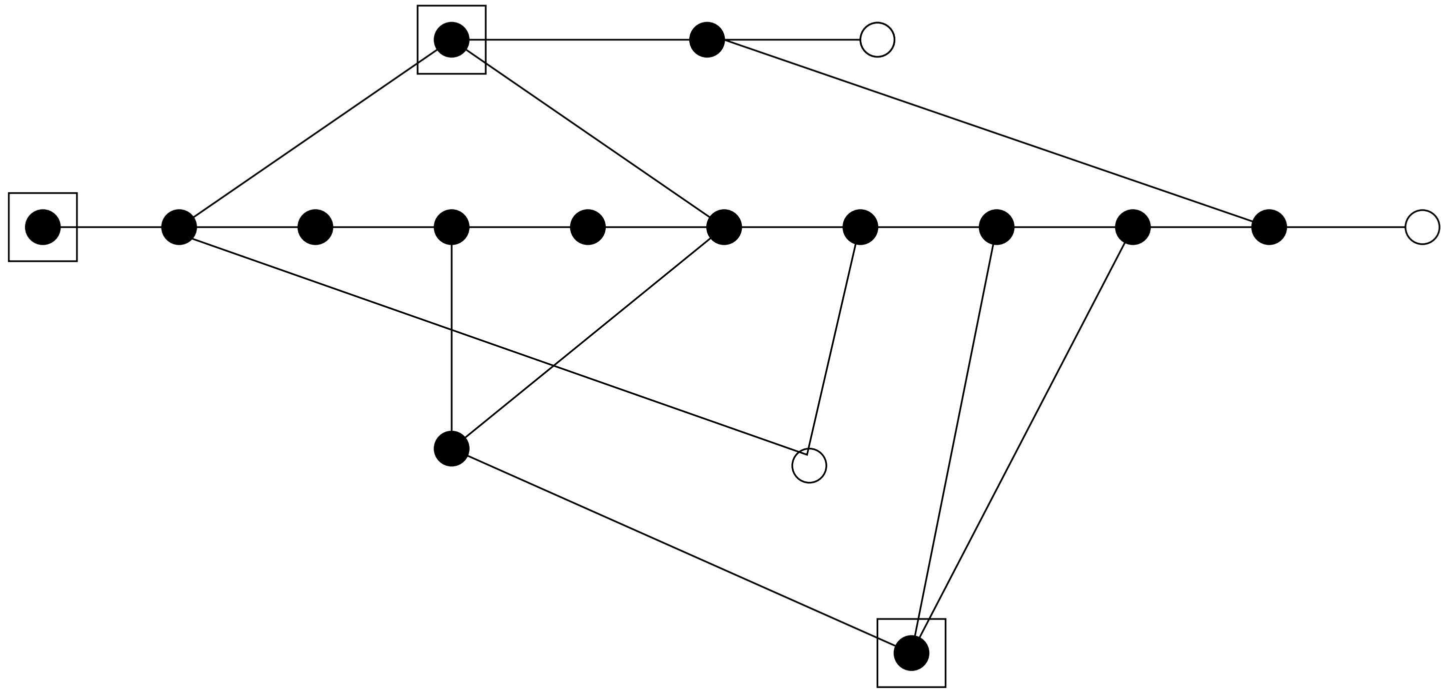
Measurement-based QC

- Teleportation Protocol (*Bennett, Brassard, Crépeau, Jozsa, Peres and Wootters*)
- Gate Teleportation (*Gottesman and Chuang)*
- One-way quantum computer *(Raussendorf and Briegel)*

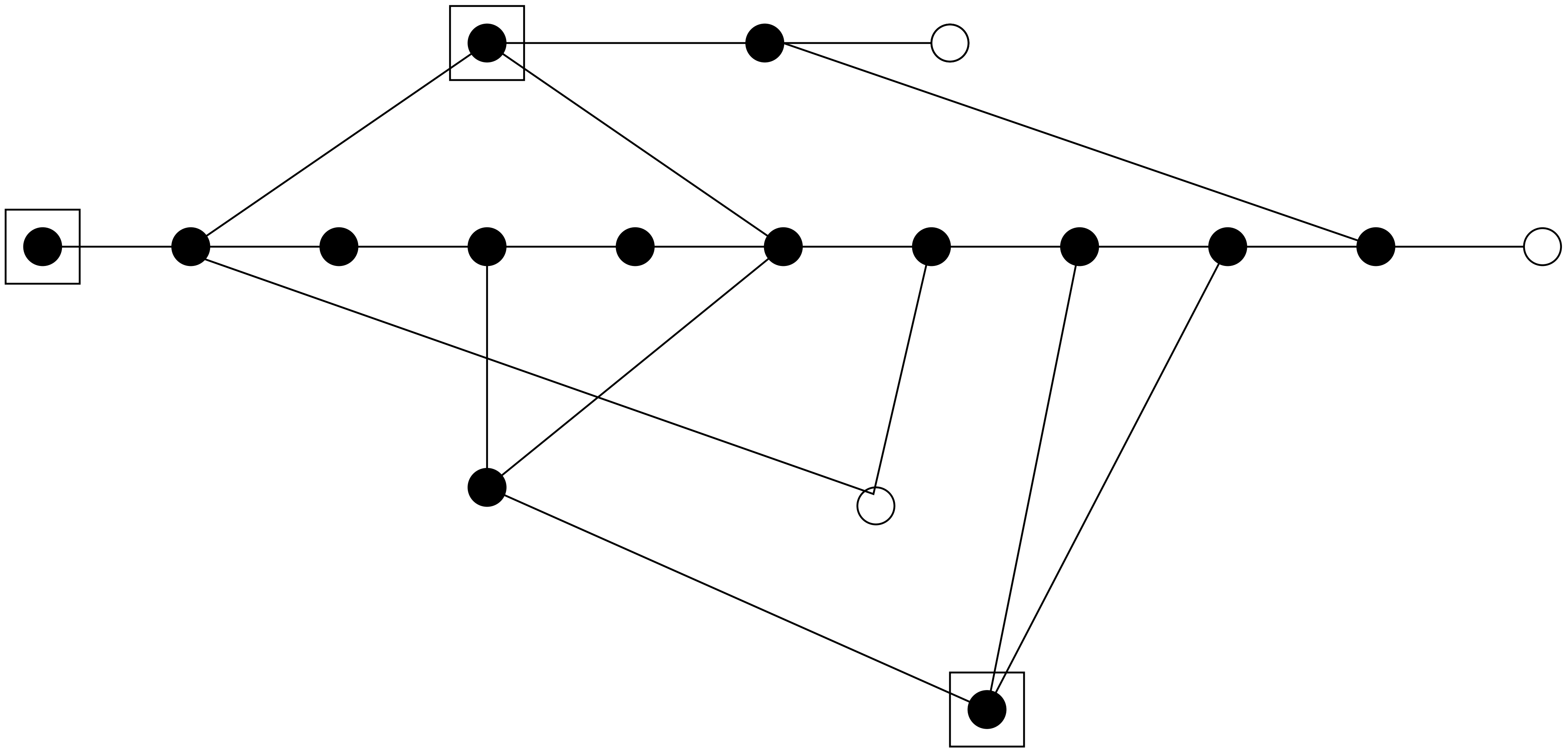
Measurements play a central role. However, measuring induces non-deterministic evolutions. This probabilistic drift can be controlled.

Elements of MBQC

- Initial entangled state (graph state)
- Angles of measurements
- Classical Control



Quantum Pacman



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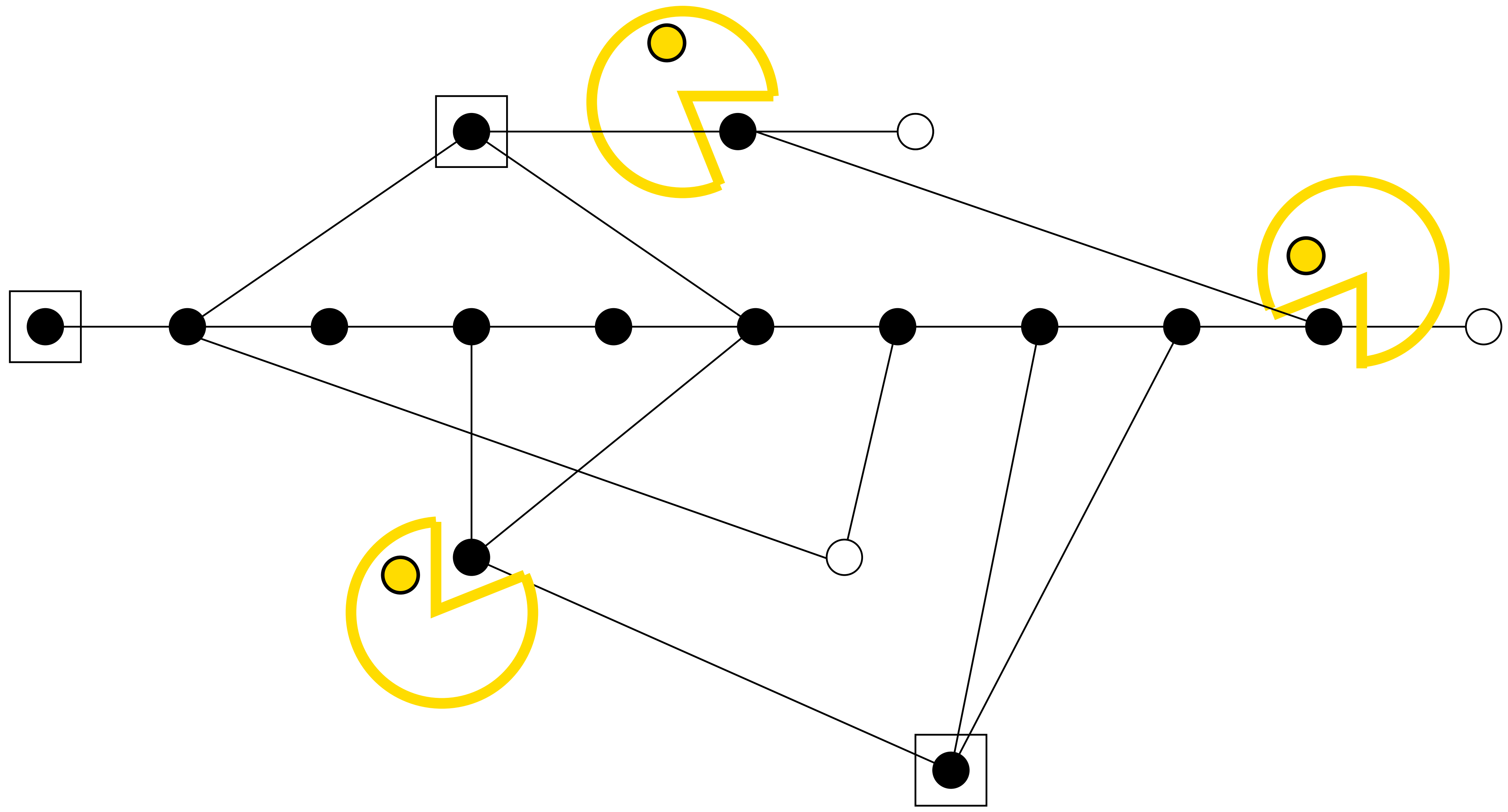
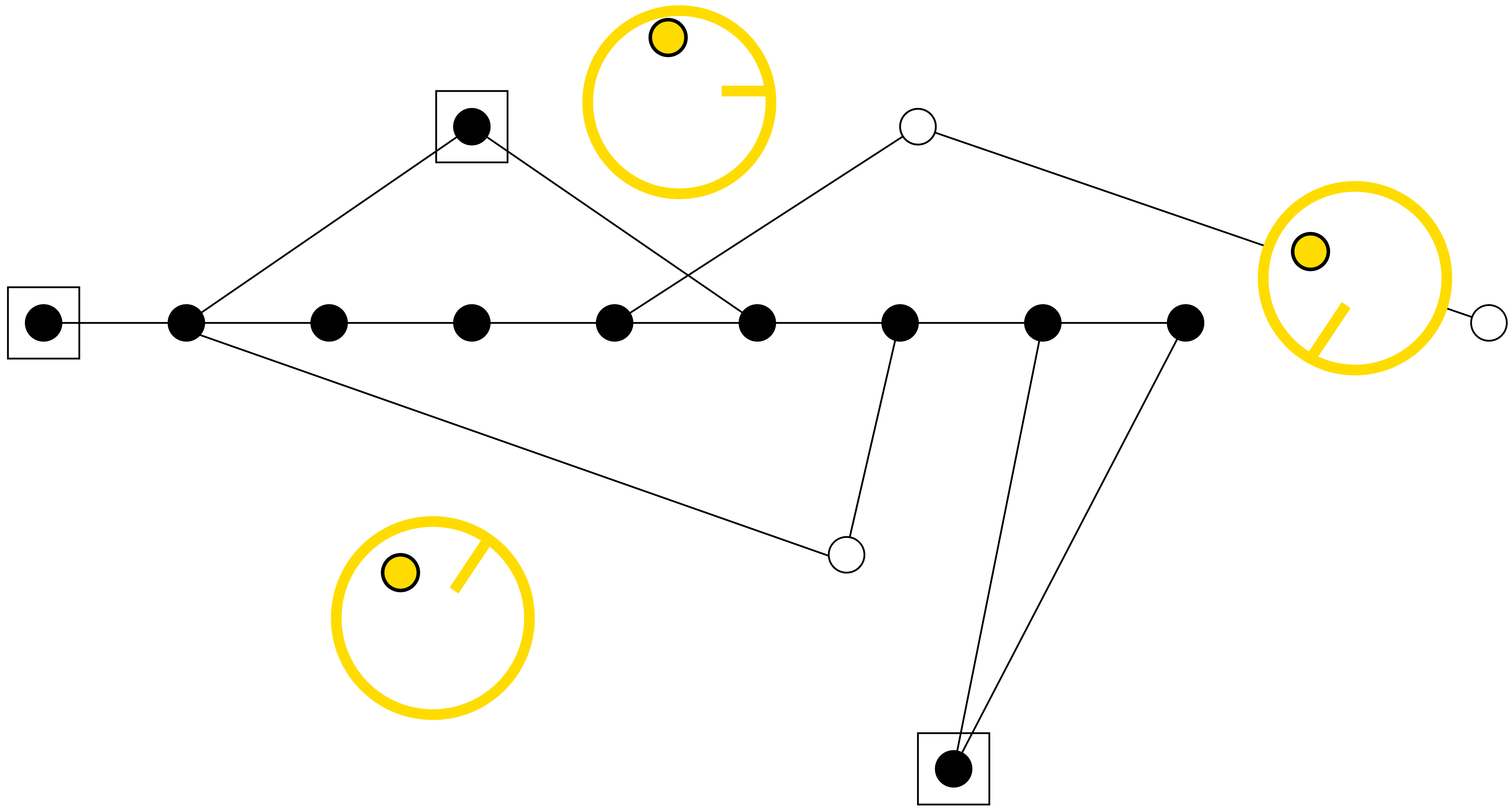
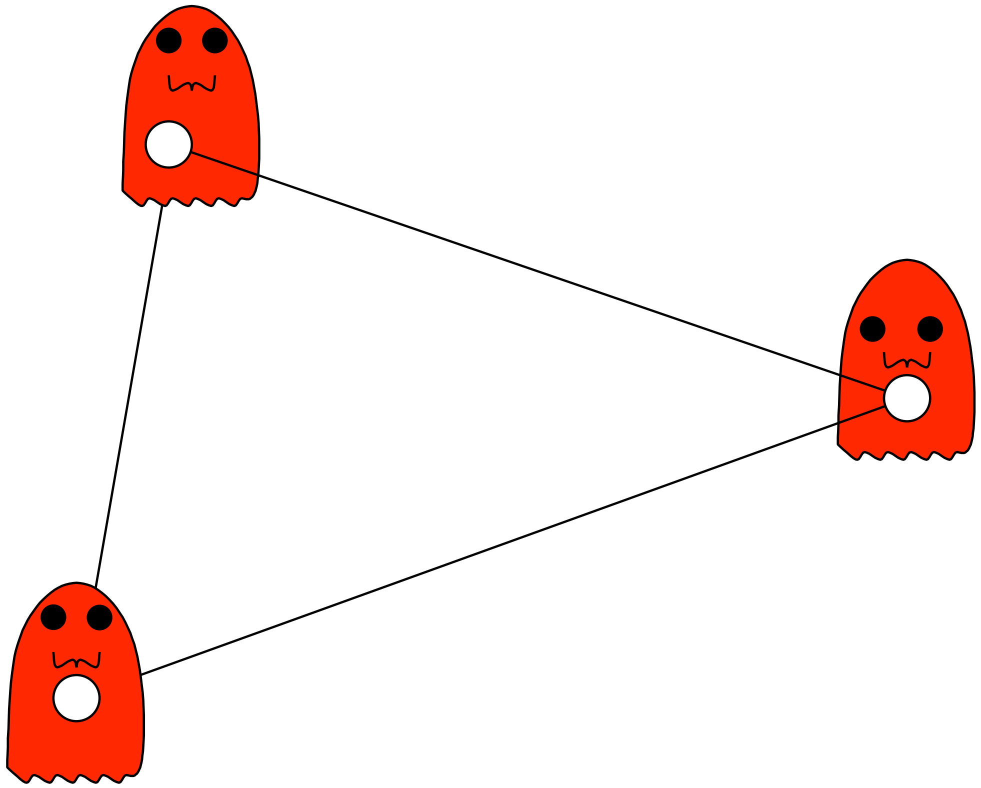


Quantum Pacman



Quantum Pacman



A formal language

- N_i prepares qubit in $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$
- M_i^α projects qubit onto basis states $\frac{1}{\sqrt{2}}(|0\rangle \pm e^{i\alpha}|1\rangle)$
  (measurement outcome is $s_i = 0, 1$)
- E_{ij} creates entanglement
- Local Pauli corrections $X_i,\ Z_i$
- **Feed forward:** measurements and corrections commands are allowed to depend on previous measurements outcomes.

$$C_i^s \qquad [M_i^\alpha]^s = M_i^{(-1)^s\alpha} \qquad {}_s[M_i^\alpha] = M_i^{\alpha+s\pi}$$

V. Danos, EK, P. Panangaden, Journal of ACM, 2007

Example

$$\mathfrak{H} := (\{1,2\},\{1\},\{2\}, X_2^{s_1} M_1^0 E_{12} N_2^0)$$

starting with the input state $(a|0\rangle + b|1\rangle)|+\rangle$ we have

$$(a|0\rangle + b|1\rangle)|+\rangle \xrightarrow{E_{12}} \frac{1}{\sqrt{2}}(a|00\rangle + a|01\rangle + b|10\rangle - b|11\rangle)$$

$$\xrightarrow{M_1^0} \begin{cases} \frac{1}{2}((a+b)|0\rangle + (a-b)|1\rangle) & s_1 = 0 \\ \frac{1}{2}((a-b)|0\rangle + (a+b)|1\rangle) & s_1 = 1 \end{cases}$$

$$\xrightarrow{X_2^{s_1}} \frac{1}{2}((a+b)|0\rangle + (a-b)|1\rangle)$$

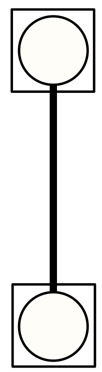
Patterns of Computation

$$(V, I, O, A_n \ldots A_1)$$

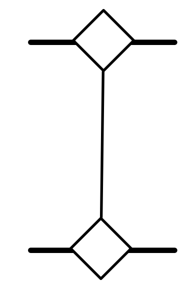
Patterns are composed sequentially or parallel

The model is universal and closed under composition

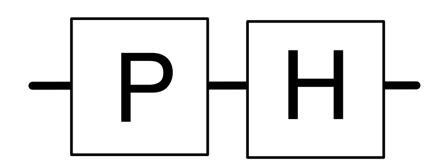
Generating Patterns



$$\wedge\mathfrak{Z} \;:=\; E_{12}$$



$$\mathfrak{J}(\alpha) \;:=\; X_2^{s_1} M_1^{-\alpha} E_{12}$$



$$\wedge Z \;:=\; \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$J(\alpha) \;:=\; \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & e^{i\alpha} \\ 1 & -e^{i\alpha} \end{pmatrix}$$

V. Danos, EK, P. Panangaden, Phys Rev. A. , 2006

MBQC vs Qcircuit

- Physical implementation, Fault Tolerance
- Equivalent in Computational Complexity
- Logarithmic separation in Depth Complexity
- **Translation forward and backward**

  ➡ Automated Scheme for Parallelising
  ➡ Information Flow
  ➡ Verification

Causal Flow - *Feed forward mechanism*

- Determinism
- Translation to Circuits
- Direct Pattern Synthesis
- Depth Complexity

Danos and Kashefi, Phys. Rev. A, 2006,
Browne, Kashefi, Mhala and Perdrix, New. Journal of Physics 2007

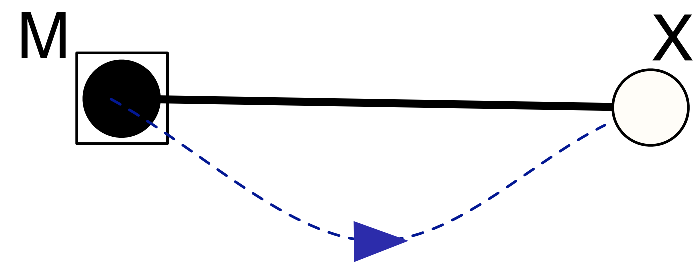
Determinism

A pattern is **deterministic** if all the branches are the same.

A necessary and sufficient condition for determinism based on **geometry** of entanglement is given by flow

Correcting Measurements

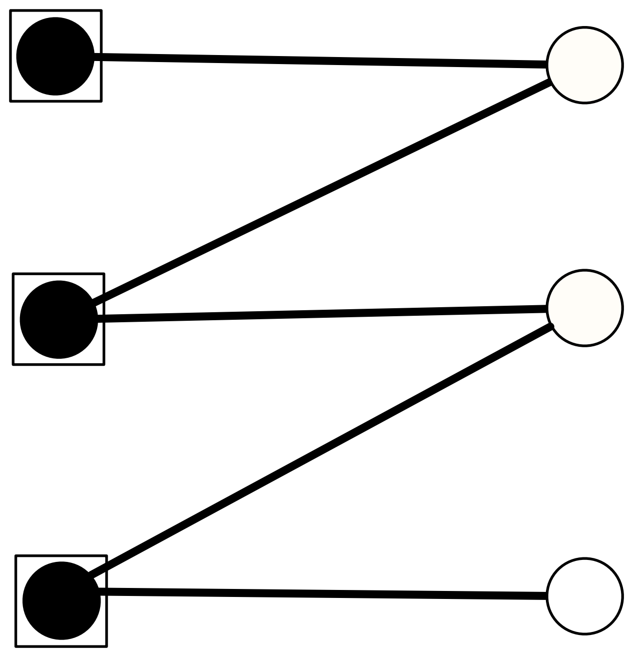
$$\mathfrak{J}(\alpha) \quad := \quad X_2^{s_1} M_1^{-\alpha} E_{12}$$



Flow

Definition. An entanglement graph (G, I, O) has flow if there exists a map $f : O^c \to I^c$ and a partial order $\preceq$ over qubits

— (i) $x \sim f(x)$
— (ii) $x \preceq f(x)$
— (iii) for all $y \sim f(x)$, we have $x \preceq y$



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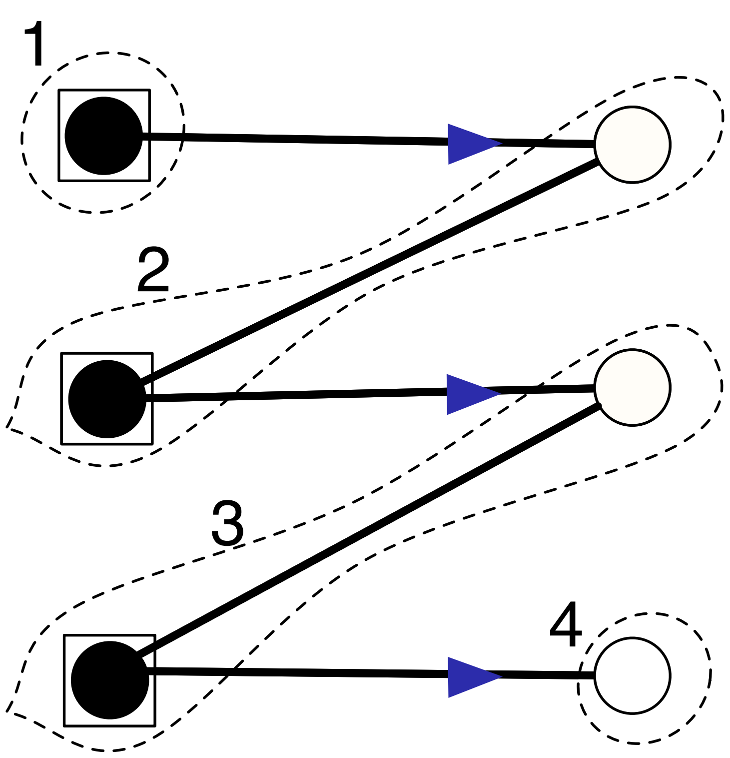


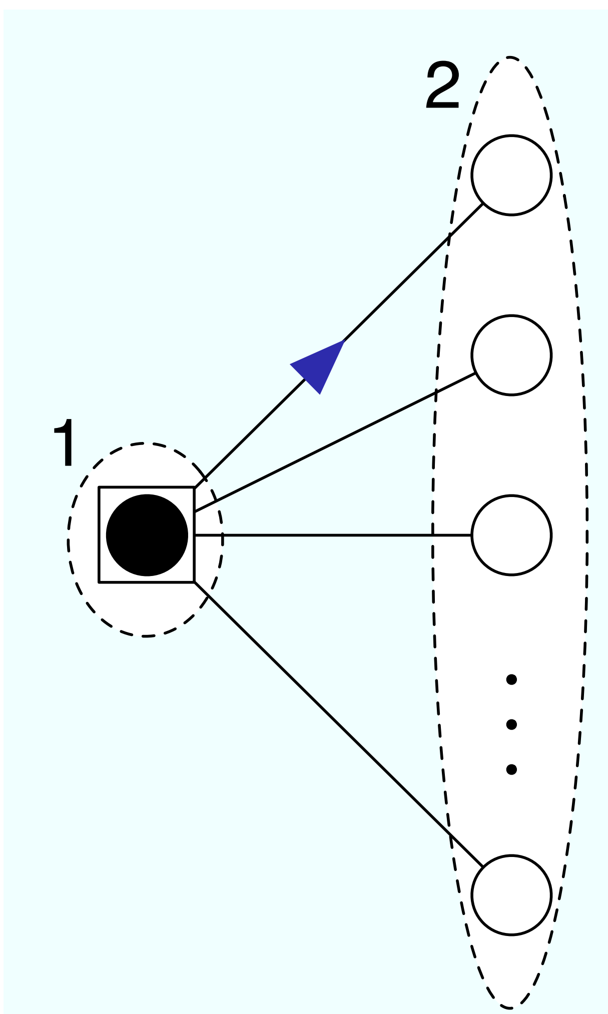
Flow

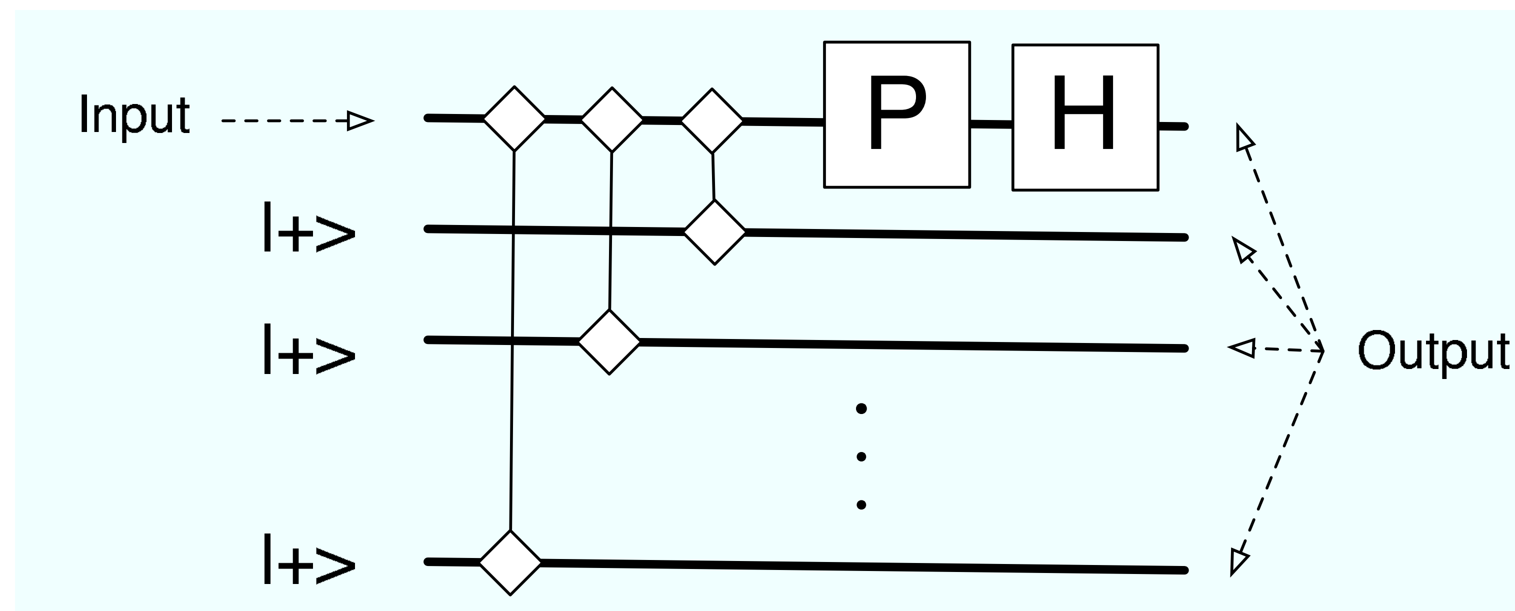
Theorem. A pattern with flow is uniformly and strongly deterministic.

Patterns with flow ⟷ Unitary embedding

From Pattern to Circuit

Star Pattern: $X_2^{s_1} M_1^{\alpha} E_{12} E_{13} \cdots E_{1n}$



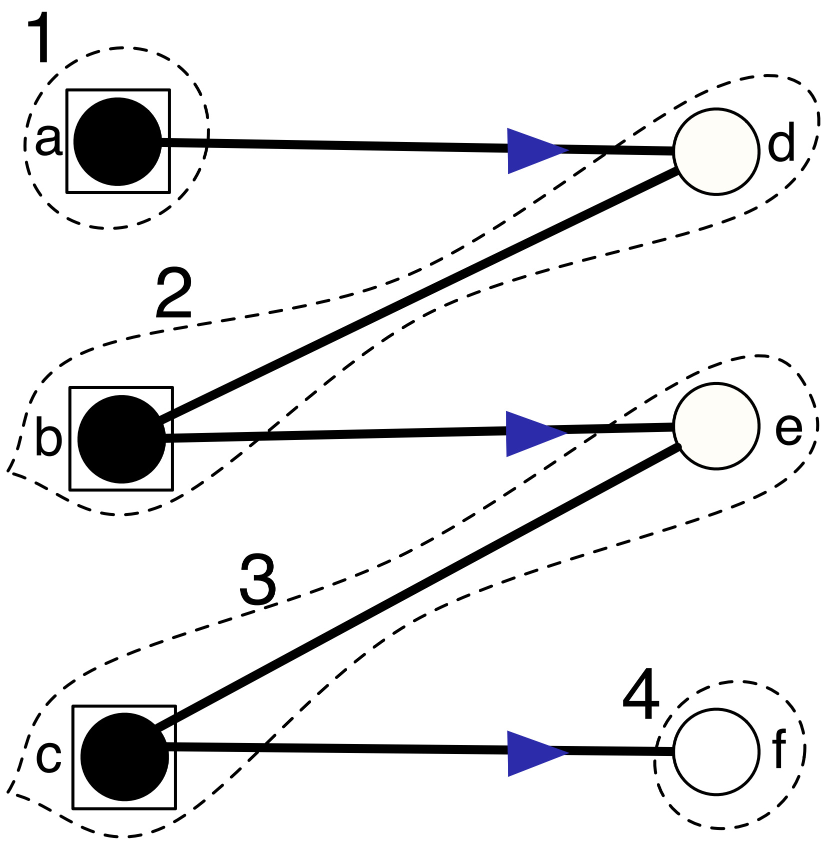


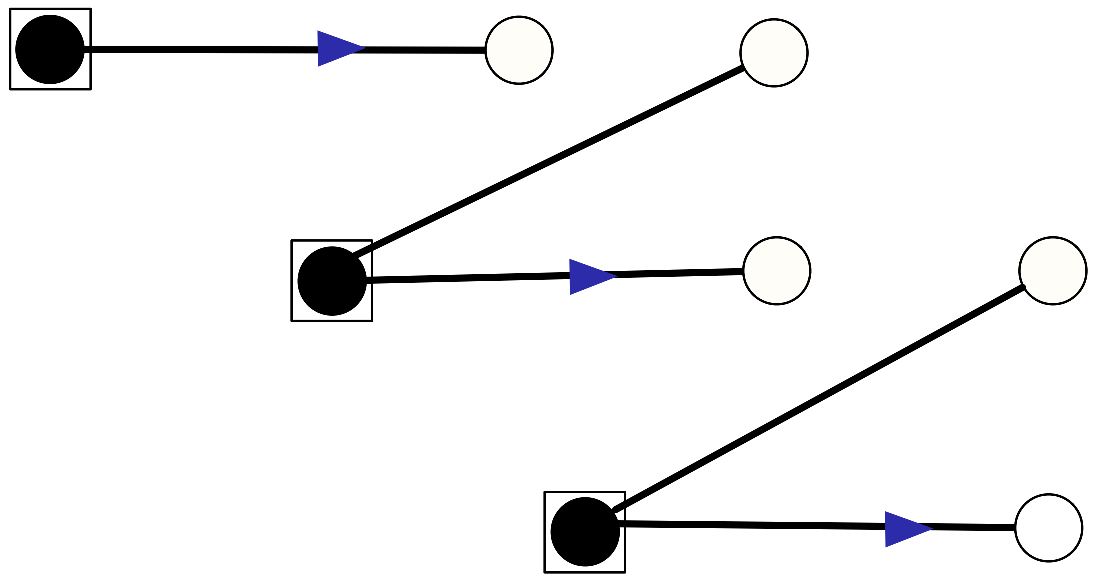
Star Decomposition

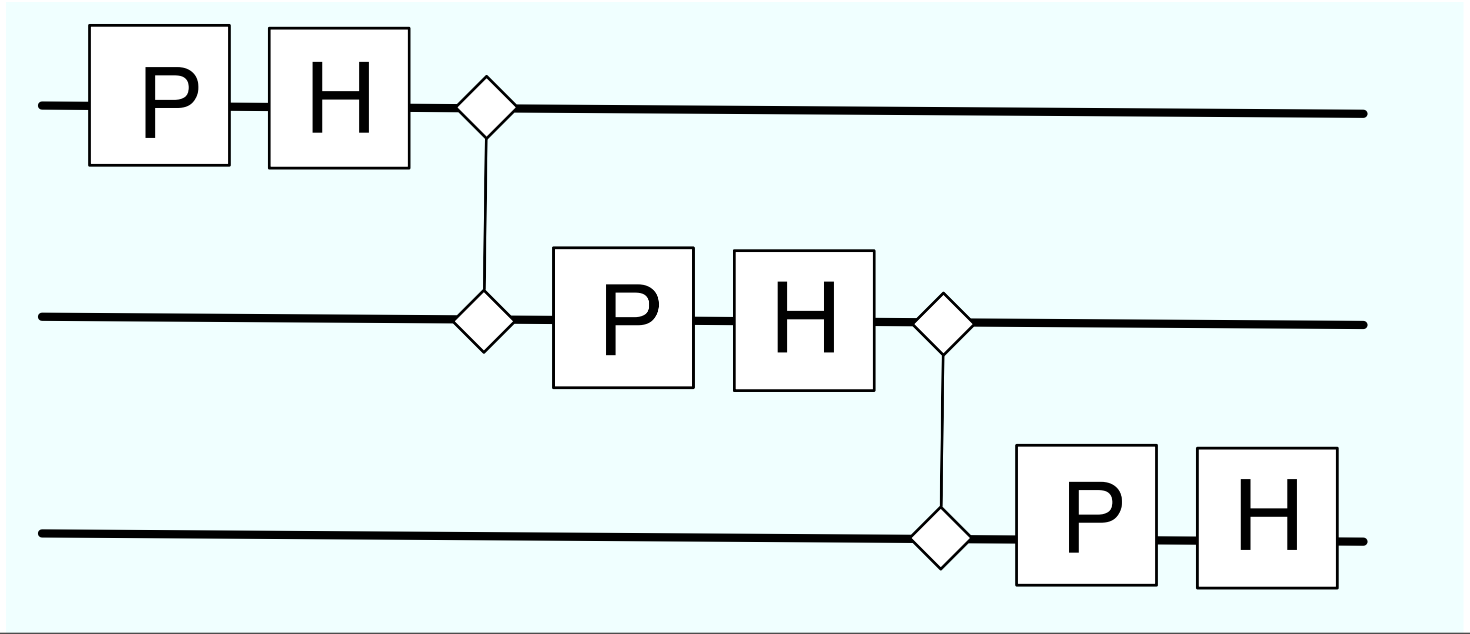
Theorem. Every pattern such that the underlying graph state has flow can be decomposed into star patterns.

Patterns with flow ⟷ Quantum Circuit

Star Decomposition

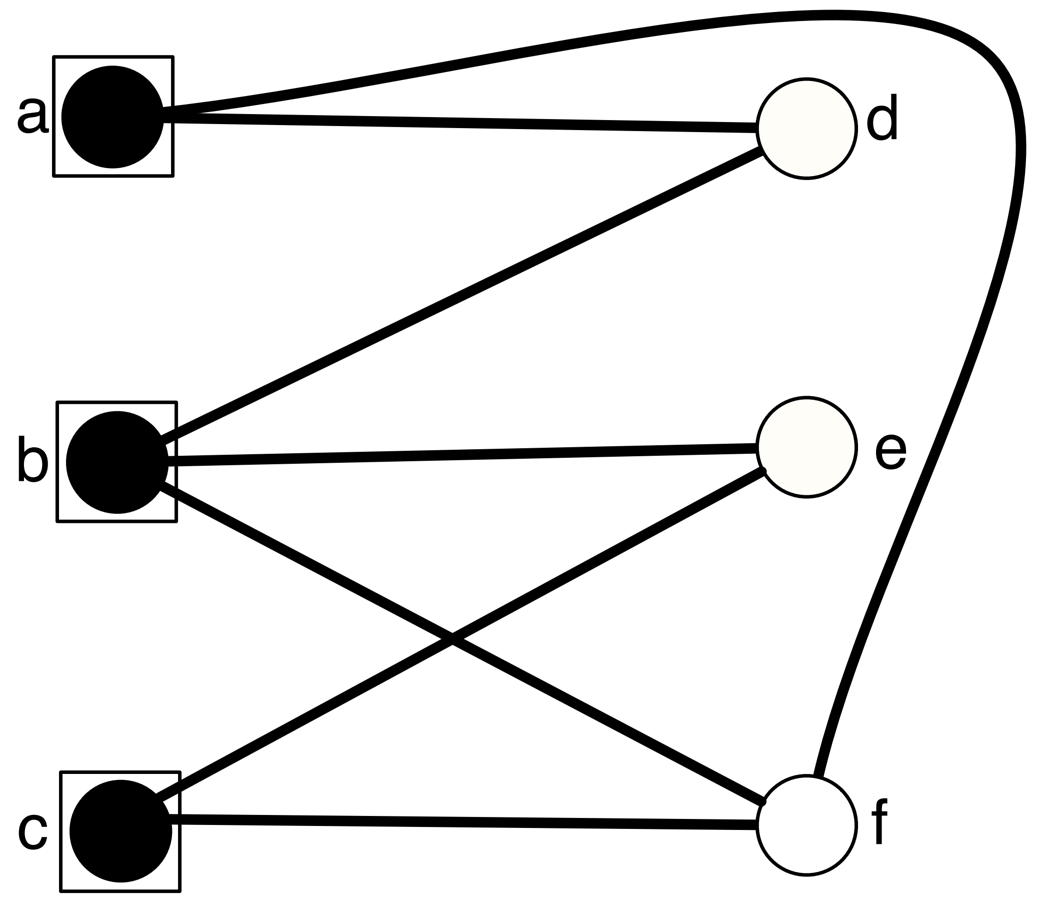






Generalised Flow

Correcting with a **set** of qubits instead of one qubit.



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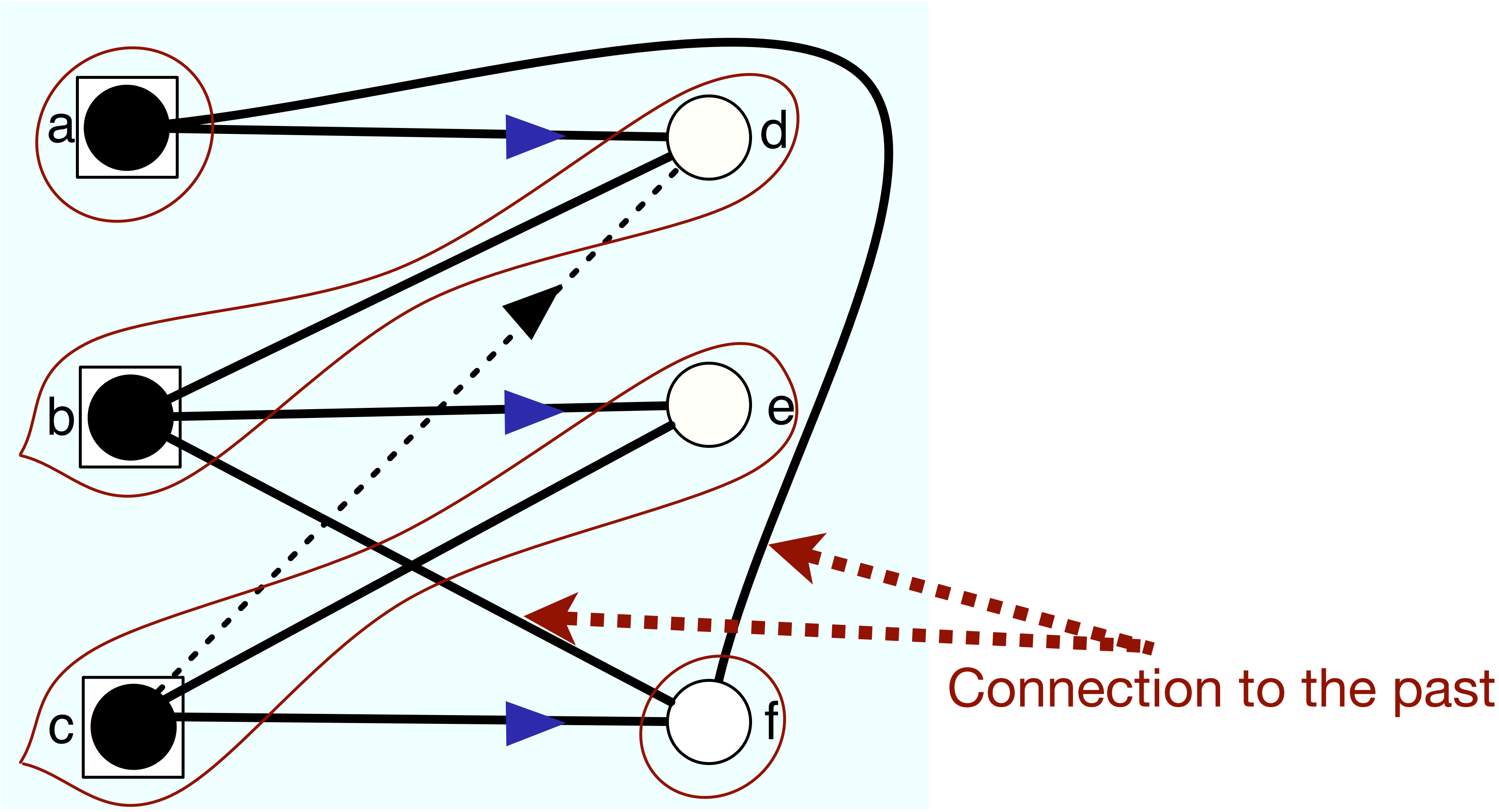


Generalised Flow

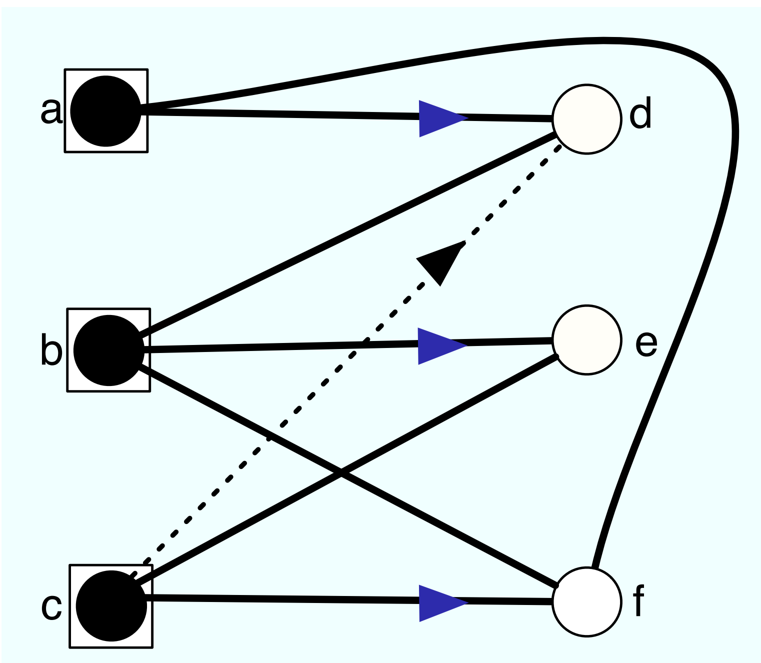
Definition. An entanglement graph (G, I, O) has generalised flow if there exists a map $f : O^c \to \mathcal{P}^{I^c}$ and a partial order $\leq$ over qubits

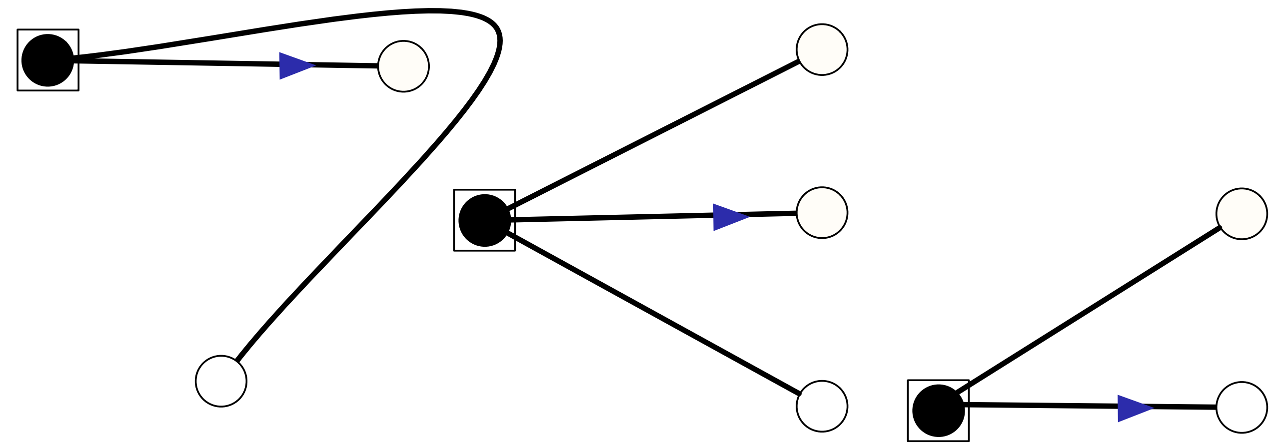
— (i) $i \notin g(i)$ and $i \in \mathrm{Odd}(g(i))$,
— (ii) if $j \in g(i)$ and $i \neq j$ then $i < j$,
— (iii) if $j \leq i$ and $i \neq j$ then $j \notin \mathrm{Odd}(g(i))$

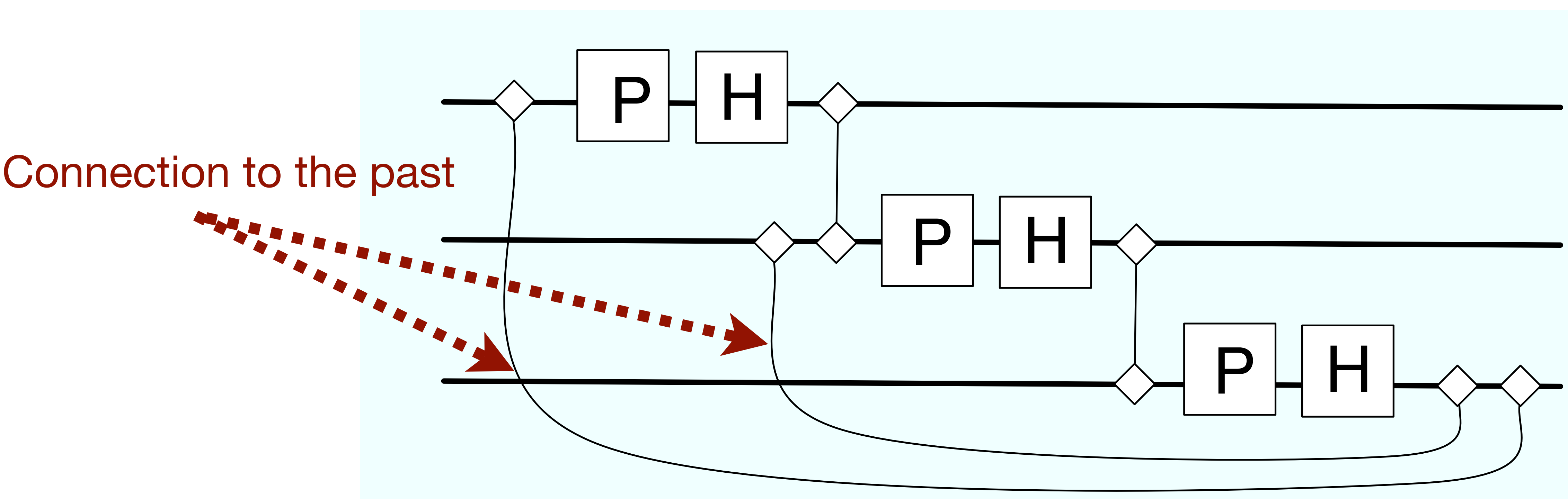
$$\mathrm{Odd}(K) = \{u\,,\ |N_G(u) \cap K| = 1 \mod 2\}$$

Theorem. A pattern is uniformly, strongly and step-wise deterministic if and only if its graph has a generalised flow.

Star Decomposition







What's the Problem ?

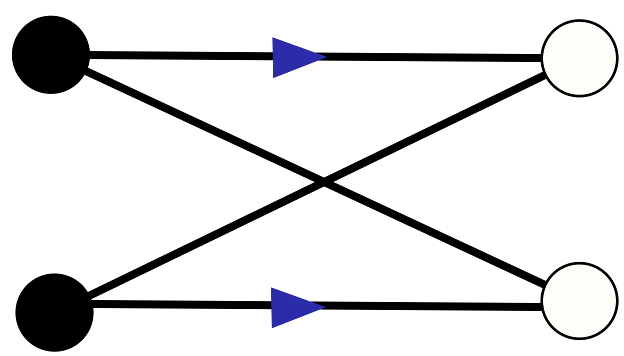
Patterns with gflow ⟷ Cyclic Quantum Circuit

Patterns with gflow ⟷ Unitary embedding

Observation. There exists a subclass of cyclic circuits implementing unitary operator !

Syntactic Characterisation

Vicious Cycle. A closed path with no two consecutive non-flow edges.

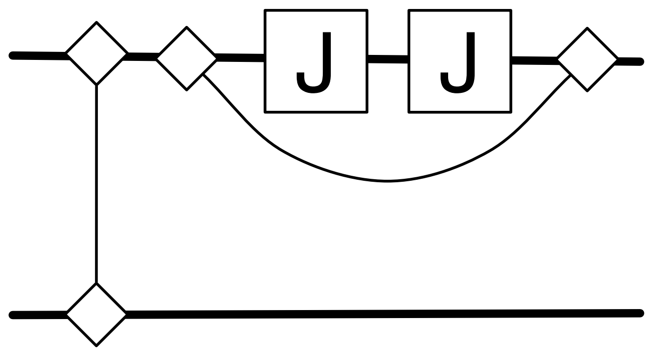


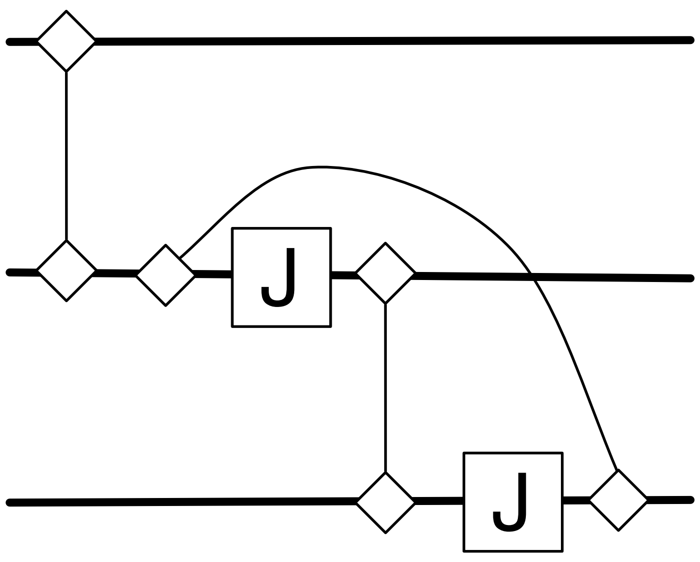
Lemma. Any gflow leads to a flow with possible vicious cycles.

Syntactic Characterisation

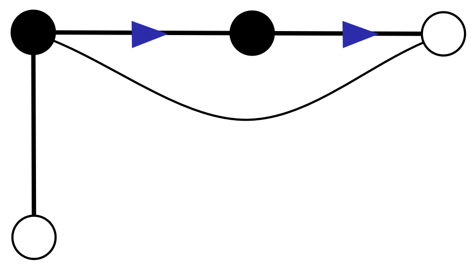
Theorem. A cyclic circuit obtained from a pattern with gflow has only following types of vicious cycles:

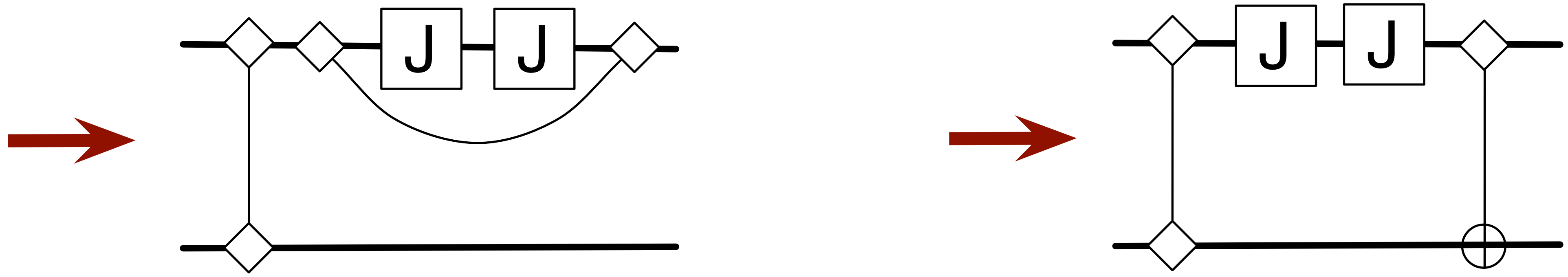
(i) Line loop (ii) Crossing loop

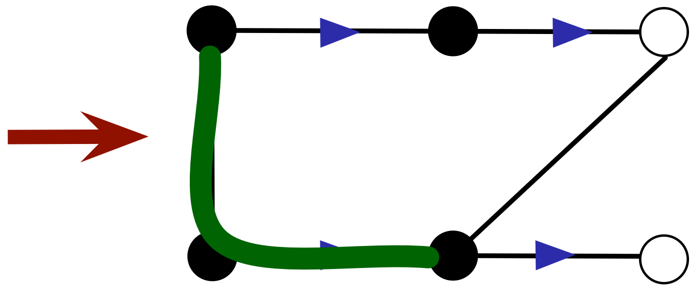




A Topological Rewriting Rule







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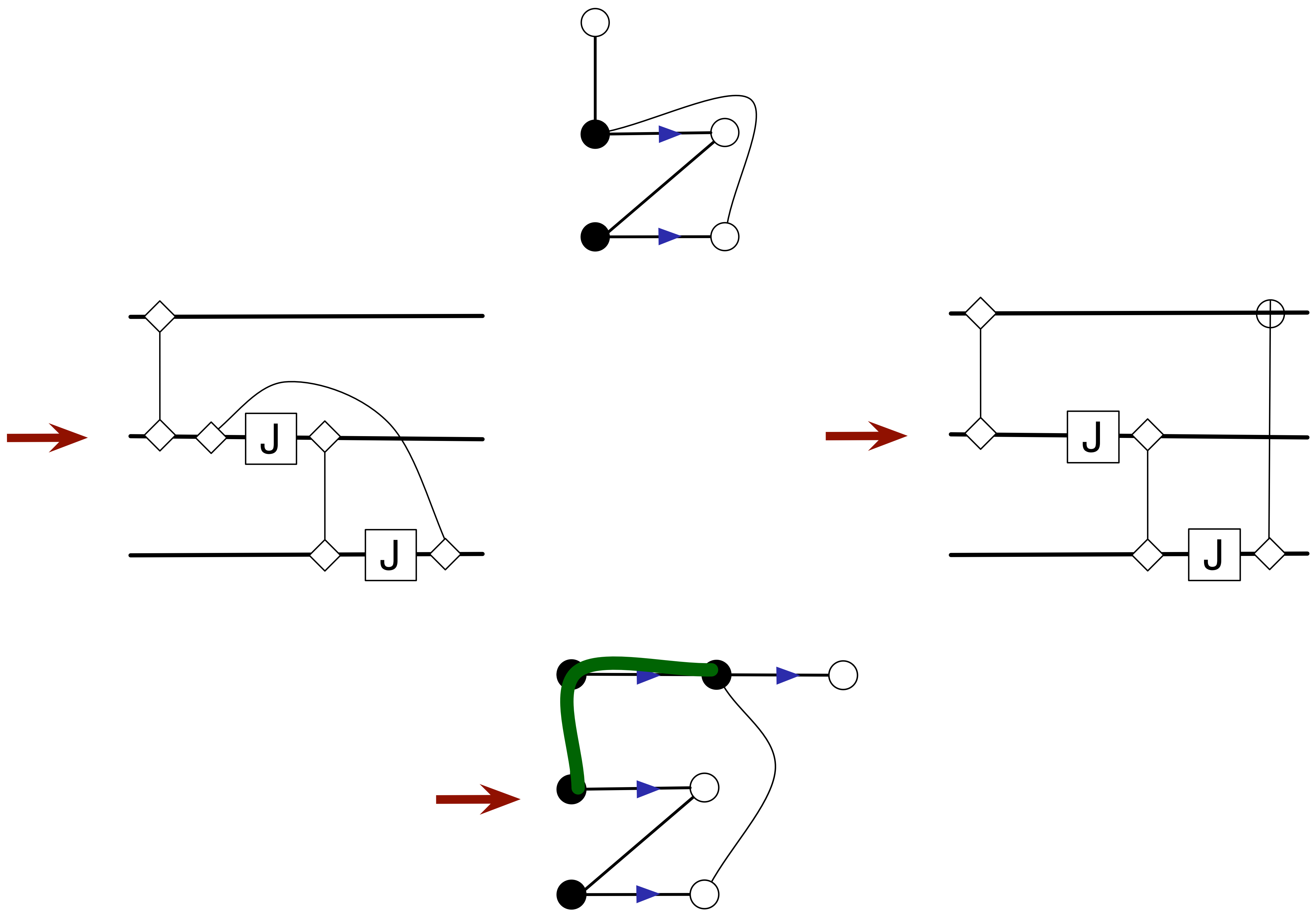
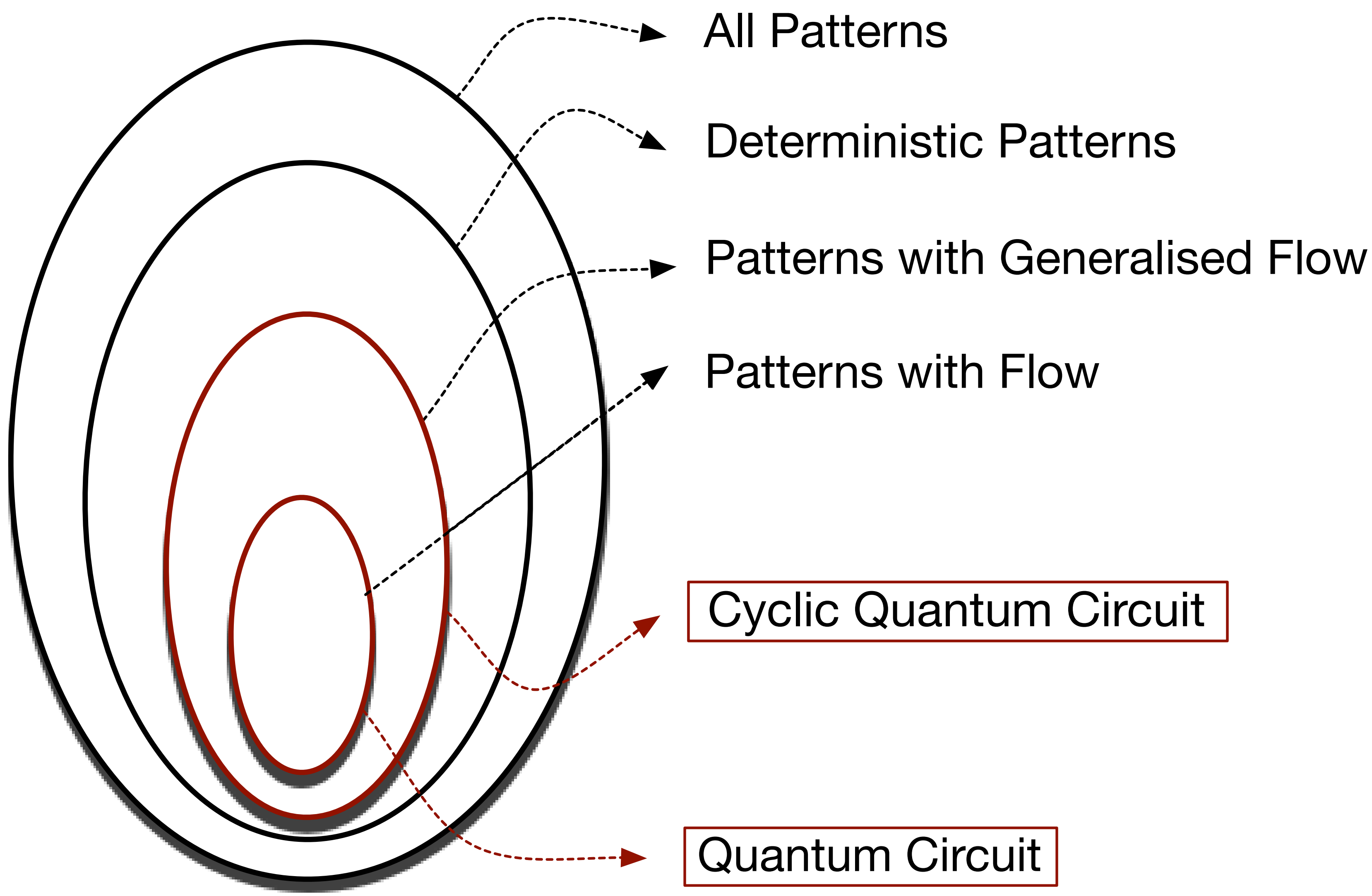


Summary- *MBQC vs Qcircuit*

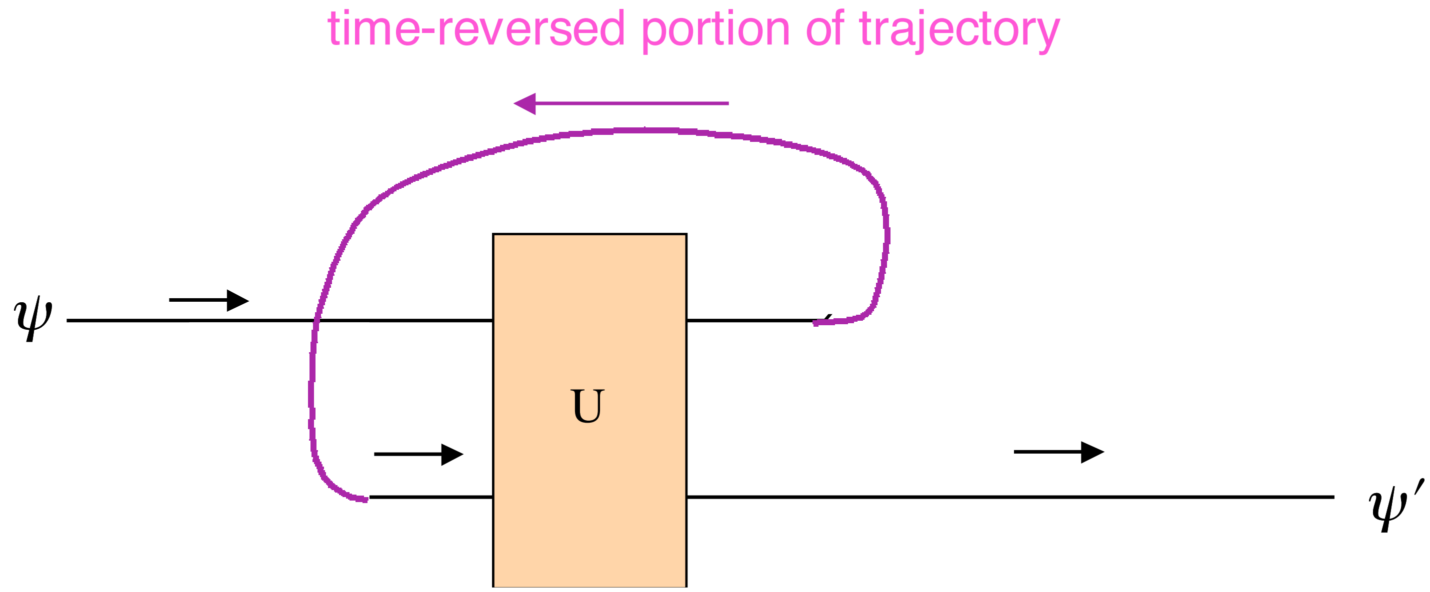


What next

- Deterministic patterns vs. quantum circuit
- Complexity analysis, exact trade off
- Physical implementation of the loop entangled state
- Connection with *timelike loop,* Deutsch, Bennett and Schumacher

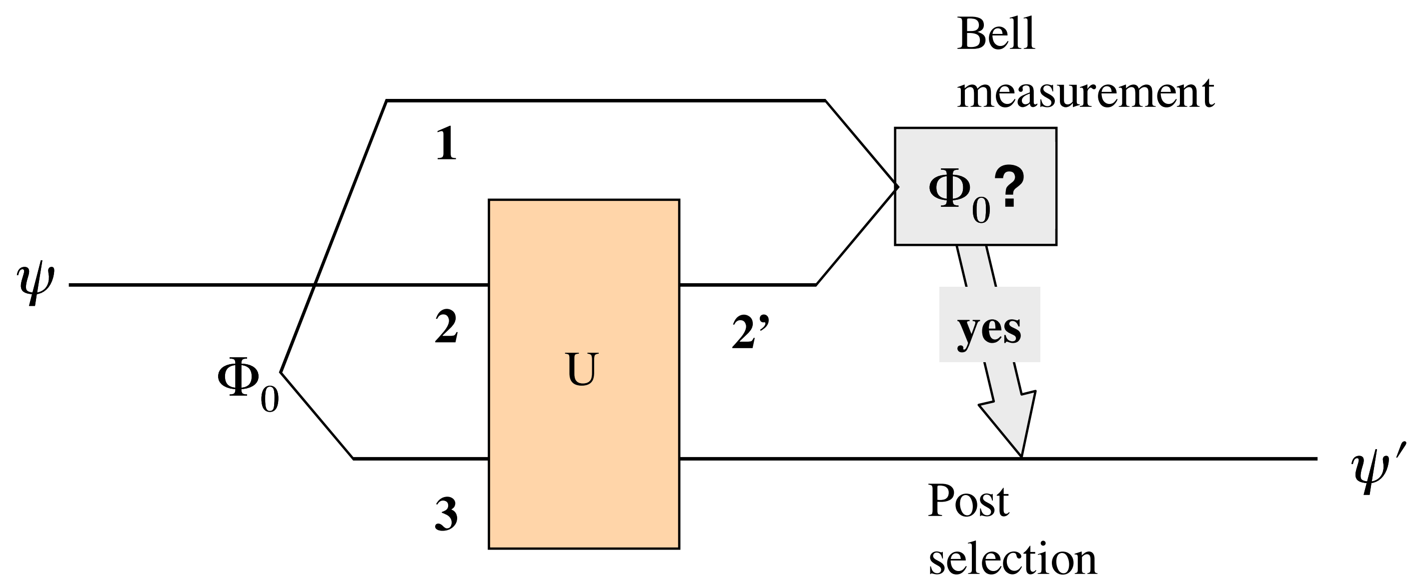
Time-like Loop *(Deutsch, Bennett and Schumacher)*

Interaction with one's past self using an exotic physical time machine !



Time-like Loop *(Deutsch, Bennett and Schumacher)*

Time-travel can be simulated using entanglement and post selection.



Computation Depth

How can we obtain a parallel algorithm for a given task?

- ➡ Depth complexity
- ➡ Fault Tolerant Implementation

MBQC. The longest feed-forward chain
QCircuit. The Layers number

Depth Complexity

All the models for QC are equivalent in computational power.

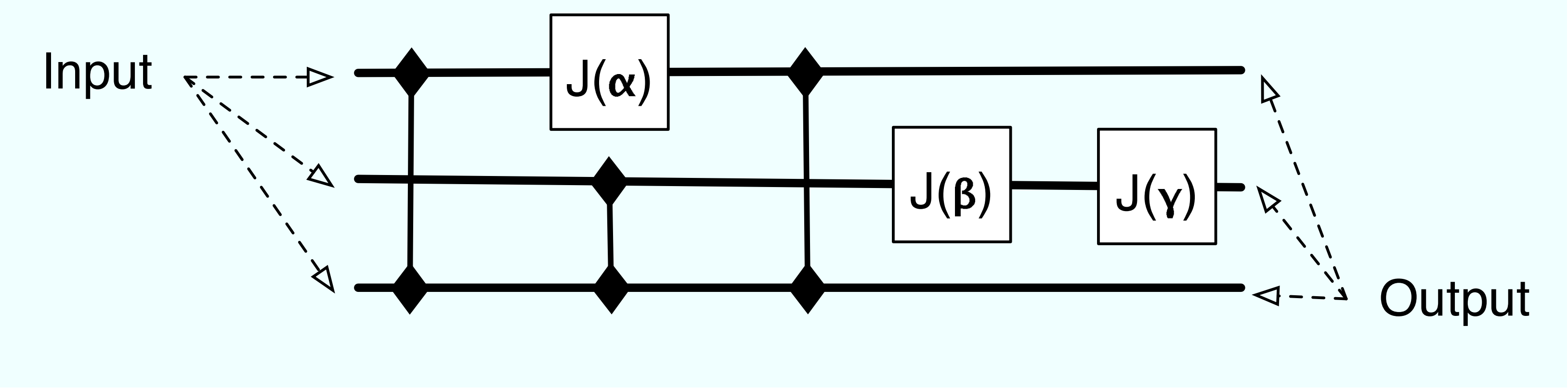
Theorem. There exists a logarithmic separation in depth complexity between MQC and circuit model.

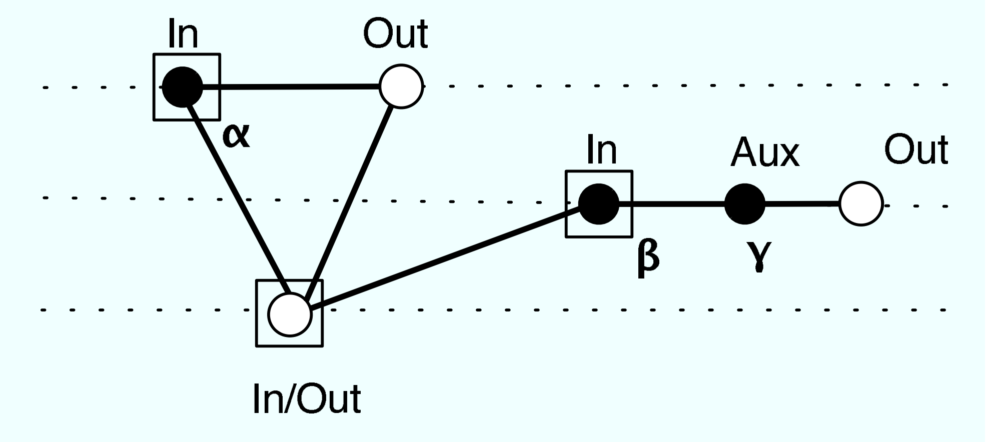
Parity function: MQC needs 1 quantum layer and $O(\log n)$ classical layers whereas in the circuit model the quantum depth is $\Omega(\log n)$

A. Broadbent and E. Kashefi, MBQC07

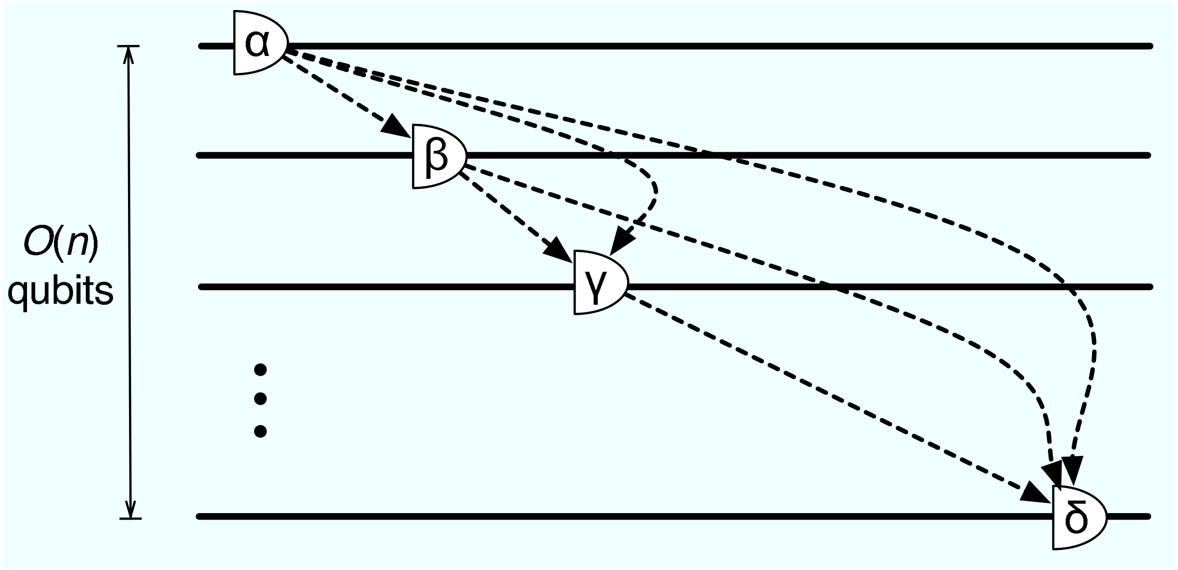
Parallelising Quantum Circuits

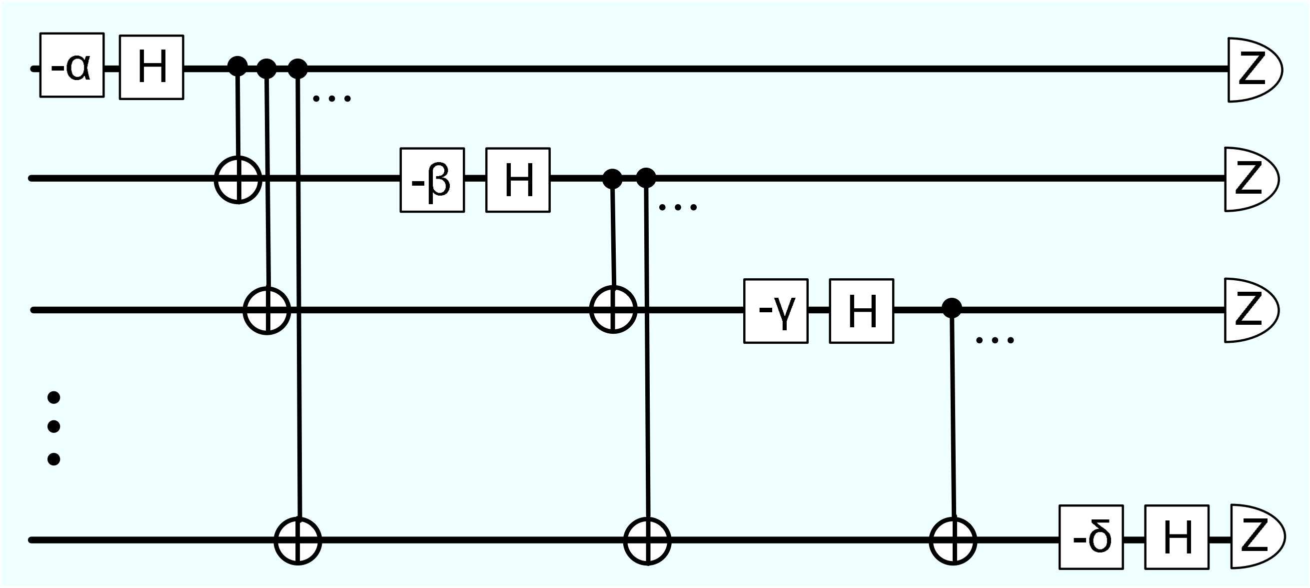
Theorem. Forward and backward translation between circuit model and MQC can only decrease the depth.





Parallelising Quantum Circuits



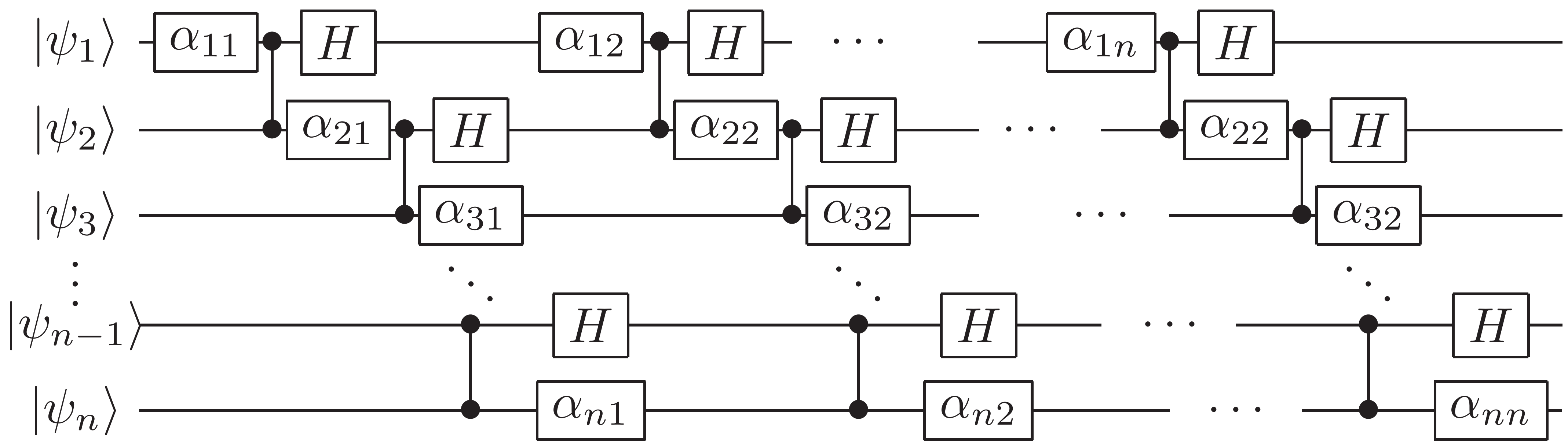


Characterisation

Theorem. A pattern has depth $d+2$ if and only if on any influencing path we obtain $P^* N^{i \leq d} P^*$ after applying the following rewriting rule:

$$N\, P_1^*\, \alpha_1 \beta_1\, P_2^*\, \alpha_2 \beta_2 \cdots P_k^*\, N \begin{cases} N N & \textit{if} \quad \forall P_i^* \neq X(XY)^* \\ N & \textit{otherwise} \end{cases}$$

Example



Can be parallelised to a pattern with depth 2