Two-Qubit Gate Operation Applied on Nearest Neighboring Neutral Atom Qubits

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Neutral Atom Quantum Computer

- Alkali atoms are used as qubits.
- Two hyperfine states of an atom span a qubit Hilbert space.
- Neutral atom is robust against external disturbance.
- A large number of atoms can be confined in an optical lattice (1-d, 2-d, and 3-d).

- One-qubit gate operation is implemented by Rabi oscillation and two-photon Raman transition.

- Two-qubit gate in a selective manner is the subject of this talk.  
Two-Qubit Gate:
Mandel et al. ’s Experiment

Controlled collisions for multi-particle entanglement of optically trapped atoms

Electro-Optic Modulator
Near-Field Fresnel Trap

Bandi *et al.*, PRA 78, 013410 (2008)

It is possible to confine an atom by a laser beam passing through a small hole whose diameter is comparable to the wave-length of the laser.

\[ a \geq \lambda_f \]
Proposal

Substrate

An array of Fresnel traps

1-d optical lattice

Near-Field Fresnel Trap

Gate control laser beam

Trap laser beam

Optical fiber

Aperture

Atom

NFFD light
Proposal

(Spatial Light Modulator with Liquid Crystal on Silicon Technology) and (Micro-Electro-Mechanical System Technology)
**Two-Qubit Gate Operation**

**Initialization**

$$|0\rangle \text{ and } |1\rangle$$

$$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

- **Step 1**
  - Gate control laser beam
  - Trap laser beam
  - Optical fiber

- **Step 2**
  - Bidirectional interaction
\[ |0\rangle, |1\rangle \]
\[ \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle) \]

- Step 3
- Step 4
- Step 5
Numerical Analysis

- $^{87}$Rb

- $|0\rangle = |F = 1, m_F = 1\rangle,$

- $|1\rangle = |F = 2, m_F = 1\rangle.$

Compatible with two-photon Raman transition

- Operations must be done adiabatically in the shortest possible time.

- Individual process fidelity = $|\langle \psi_0 | \psi(t = T) \rangle|$

  Overlap between the ground state of the final potential and the time-evolved wave function

  $= 0.99$
### Parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Numerical values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radius of an aperture</td>
<td>$a = 1.5 \lambda_F \approx 1.2 \mu m$</td>
</tr>
<tr>
<td>Wavelength of the trap laser beam</td>
<td>$\lambda_F = 795.118 \text{ nm}$</td>
</tr>
<tr>
<td>Laser intensity</td>
<td>$I = 2.5 \times 10^5 \text{ W/cm}^2$</td>
</tr>
<tr>
<td>Wavelength of the optical lattice</td>
<td>$\lambda_{OL} = 785 \text{ nm}$</td>
</tr>
<tr>
<td>D$_1$ transition of $^{87}\text{Rb}$</td>
<td>$\lambda_0 = 794.979 \text{ nm}$</td>
</tr>
<tr>
<td>Depth of trapping potential at t=0</td>
<td>$U_0 = h \times 1.03 \times 10^6 \text{ Hz}$</td>
</tr>
<tr>
<td>Depth of the optical lattice potential at t=0</td>
<td>$V_0 = h \times 1.47 \times 10^5 \text{ Hz}$</td>
</tr>
<tr>
<td>Minimum of NFFD potential</td>
<td>$z_m \approx 1.7 \mu m$</td>
</tr>
</tbody>
</table>
Fresnel Trap Potential

Rayleigh-Sommerfeld formula

\[ U_F(x, y, z) = -U_0 \frac{\mid \epsilon(x, y, z) \mid^2}{E_0^2} \]

\[ U_0 = \frac{3}{8} \frac{\gamma E_0^2}{|\delta| k^3} \]

\[ \epsilon(x, y, z) = \frac{E_0}{2\pi} \iint \frac{e^{ikr}}{r} \left( \frac{z}{r} \right) \left( \frac{1}{r} - ik \right) dx' dy' \]
Optical Lattice Potential

\[ U_{OL}(x) = -V_0 \cos^2(2k_{OL} x) e^{-2(y^2 + (z-z_m)^2)/w^2} \]
Step 1

\[ U_F(\varphi, t) = \cos^2 \left( \frac{\pi}{2T_1} t \right) U_F(\varphi) \]

\[ T_{FT} = \text{Time required to attain fidelity } 0.99 \]
\[ = 1.8 \text{ ms} \]
Step 2

- Optical lattice potential which is produced by a pair of counterpropagating laser beams which have different polarizations.

\[ E_+(x) + E_-(x) \propto \sigma^+ \cos(kx - \theta) + \sigma^- \cos(kx + \theta) \]

- Optical potentials

\[ V_\pm(x) \propto \cos^2(kx \mp \theta) \]

\[ \theta(t) = n\pi \sin^2 \left( \frac{\pi}{2T_{OL}} t \right) \]

- Effective potential for each hyperfine state

\[ V_{|0\rangle}(x) = \frac{1}{4} V_+(x) + \frac{3}{4} V_-(x), \]

\[ V_{|1\rangle}(x) = \frac{3}{4} V_+(x) + \frac{1}{4} V_-(x). \]
\[ i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \psi + V_{1\rangle}(t) \psi \]

\[ T_{OL} = \text{Time required to attain fidelity 0.99} = 1.28 \text{ ms} \]
Step 3

- Interaction energy

\[
U_{\text{int}} = g \int |\psi|^4 dx \quad \text{where} \quad g = 4\pi\hbar^2 a/m
\]

- \(U_{\text{int}}t_{\text{hold}} = \pi\)

- Two-qubit gate

\[
U = |00\rangle\langle 00| + e^{-i\pi} |01\rangle\langle 01| + |10\rangle\langle 10| + |11\rangle\langle 11|
\]

\[
= \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
\]

\[
T_{\text{Int}} = 2.29 \text{ ms}
\]
Step 4 and Step 5

Atoms are returned to their initial positions in the optical lattice (reverse of Step 2) and then NFFD traps are switched on (inverse of Step 1).

Fidelity of spectator atom during the two-qubit gate operation
Execution Time and Fidelity

- Overall execution time

\[ T_{\text{overall}} = 2(T_{\text{FT}} + T_{\text{OL}}) + T_{\text{int}} \approx 8.45 \text{ ms.} \]

- Overall fidelity

\[ 0.99^8 = 0.923 \]
Summary

• Selective two-qubit gate operation is possible if atoms are trapped in an optical lattice generated by near field Fresnel diffraction of light at an aperture of variable size.
• We proposed an experiment towards the demonstration of a selective two-qubit gate operation.
• We have obtained an upper bound of the gate operation time of 8.45 ms with corresponding fidelity of 0.923.
• Reduction of the gate operation time in is possible by increasing the laser intensity.
• The proposal should be feasible within existing technology.
Thank you for your attention
AC Stark Shift (Light Shift)

\[ E(x, t) = \Re(E_0(x)e^{-i\omega_L t}) \]

\[ H_i = -\frac{1}{2}(E_0 \cdot d)(e^{-i\omega_L t} + e^{i\omega_L t}) \quad |\psi\rangle = c_g |g\rangle + c_e |e\rangle \]

\[ V(x) = \frac{\hbar |\Omega_{eg}(x)|^2}{4 (\Delta_{eg})} \]

\[ \Omega_{eg} = \langle e | d | g \rangle \cdot E_0(x)/\hbar \quad \Delta_{eg} = \omega_L - \omega_0 \]

\[ \omega_L \]  

Blue-detuned

\[ \omega_0 \]  

Red-detuned
One-Qubit Gate: 
Mandel et al’s Proposal

One-qubit gate implementation by MW pulses

\(^{87}\text{Rb}\)

\[ |0\rangle = |F = 1, m_F = -1\rangle = -\frac{\sqrt{3}}{2} |\frac{3}{2}, -\frac{3}{2}\rangle |\frac{1}{2}, \frac{1}{2}\rangle + \frac{1}{2} |\frac{3}{2}, -\frac{1}{2}\rangle |\frac{1}{2}, -\frac{1}{2}\rangle, \]

\[ |1\rangle = |F = 2, m_F = -2\rangle = \frac{3}{2} |\frac{3}{2}, -\frac{3}{2}\rangle |\frac{1}{2}, -\frac{1}{2}\rangle. \]
**One-Qubit Gate Operation**

One-qubit operation by two-photon Raman transition (site selective addressing)

- Detuning $\hbar \Delta = \hbar \omega_L - (E_e - E_0)$.
- Rabi Osc. $\Omega_i : |i\rangle \leftrightarrow |e\rangle$.
- General state
  
  \[ |\psi\rangle = c_0 |01\rangle + c_1 |11\rangle + c_e |e0\rangle. \]
  
  ($|ik\rangle = |i\rangle \otimes |\text{photon} \neq k\rangle$)

\[
\begin{align*}
  i \frac{d}{dt} \begin{pmatrix} c_0 \\ c_1 \end{pmatrix} &= H \begin{pmatrix} c_0 \\ c_1 \end{pmatrix} \\
  H &= \frac{1}{2} \epsilon \sigma_z - \frac{\Omega_0 \Omega_1}{4\Delta} \sigma_x.
\end{align*}
\]

Controllable $\epsilon, \Omega_i, \Delta$.

\[
\epsilon = E_1 - E_0 + \frac{\Omega_1^2}{4\Delta} - \frac{\Omega_0^2}{4\Delta}
\]

$i\sigma_x, i\sigma_z$ are generators of $\mathfrak{su}(2)$. 

(Shown is a diagram of a Raman transition with energy levels $E_0$, $E_e$, and $E_1$, and Rabi frequencies $\Omega_0$ and $\Omega_1$.)
State Selective Potential

\[ m_J = \frac{3}{2} \quad m_J = \frac{1}{2} \quad m_J = -\frac{1}{2} \quad m_J = -\frac{3}{2} \]

\[ \sigma^+ \quad \sigma^- \]

\[ nP_{3/2} \quad nP_{1/2} \quad nS_{1/2} \]

\[ E_+ \propto e^{ikx}(\hat{z}\cos \theta + \hat{y}\sin \theta) + E_- \propto e^{-ikx}(\hat{z}\cos \theta - \hat{y}\sin \theta) \]

\[ E_+ + E_- \propto \sigma^+ \cos(kx - \theta) + \sigma^- \cos(kx + \theta). \]
Optical lattice potentials

\[ V_\pm(x) \propto \cos^2(kx \mp \theta) \quad \theta \text{ is controllable.} \]

Effective potential for each hyperfine state

\[ |0\rangle = |F = 1, m_F = 1\rangle = \frac{\sqrt{3}}{2} \left| \frac{3}{2}, \frac{3}{2}\right\rangle \left| \frac{1}{2}, -\frac{1}{2}\right\rangle - \frac{1}{2} \left| \frac{3}{2}, \frac{1}{2}\right\rangle \left| \frac{1}{2}, \frac{1}{2}\right\rangle, \]

\[ |1\rangle = |F = 2, m_F = 1\rangle = \frac{\sqrt{3}}{2} \left| \frac{3}{2}, \frac{1}{2}\right\rangle \left| \frac{1}{2}, \frac{1}{2}\right\rangle + \frac{1}{2} \left| \frac{3}{2}, \frac{3}{2}\right\rangle \left| \frac{1}{2}, -\frac{1}{2}\right\rangle. \]

\[ \rightarrow V|0\rangle(x) = \frac{1}{4} V_+(x) + \frac{3}{4} V_-(x), \quad V|1\rangle = \frac{3}{4} V_+(x) + \frac{1}{4} V_-(x). \]
Electro-Optic Modulator