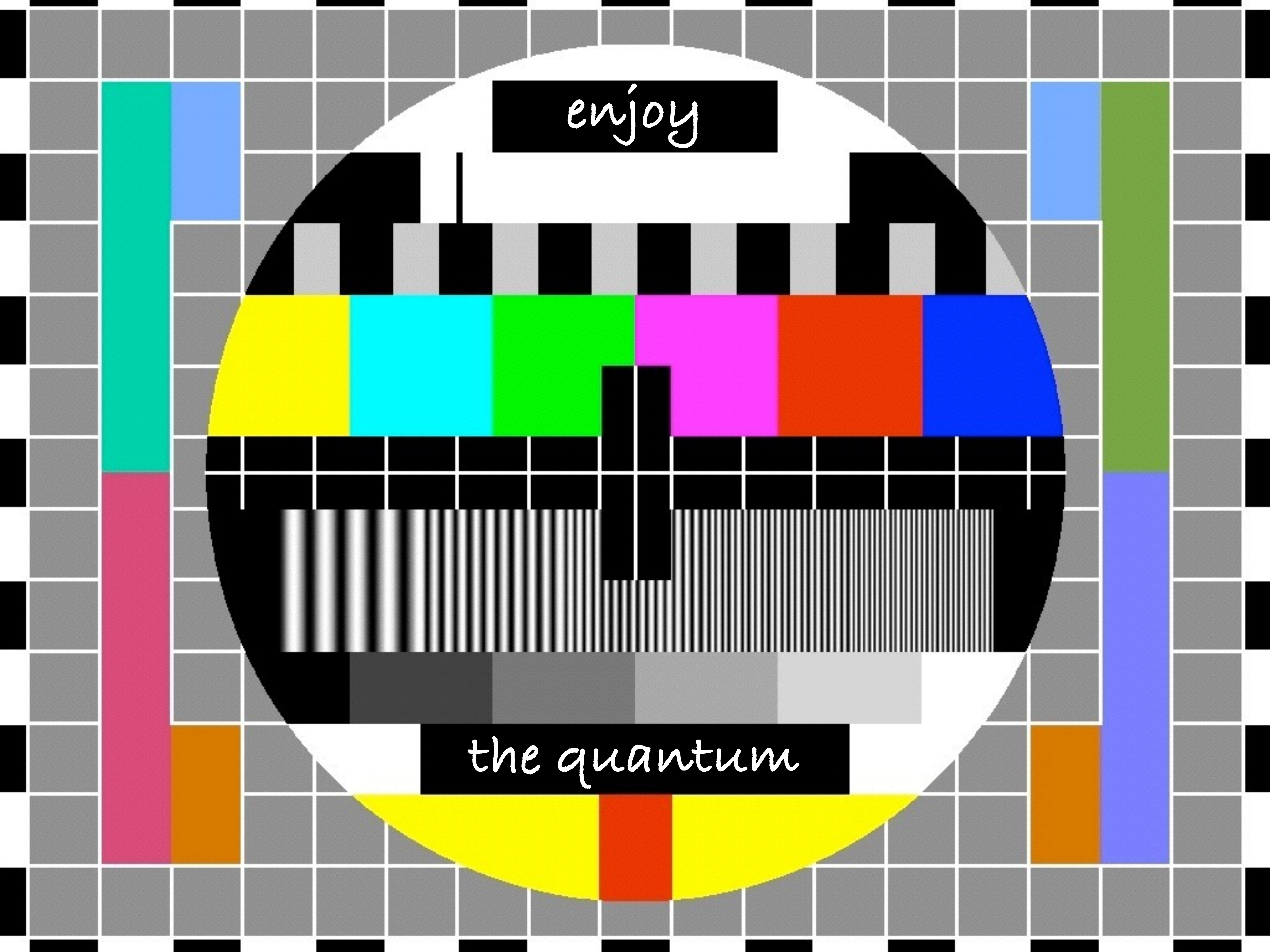

enjoy
the quantum

Quantum estimation, sensing, metrology, probing and the like

Matteo G. A. Paris
Quantum Technology Lab
Dipartimento di Fisica “Aldo Pontremoli”
Università degli Studi di Milano, Italy

Iran International Conference on Quantum Information (IICQI-20)

“Here”, Thursday October 22th, 2020

What we are going to speak about?

ϱ_λ

$$\{\Pi_x\}_{x\in\mathcal{X}}$$

$$\chi = (x_1, x_2, \ldots)$$

- Optimal measurements
- Ultimate bounds to precision

Quantum estimation, sensing and metrology

- Motivations (fundamental and applicative) + basic ideas about estimation
- Classical estimation theory
- Quantum estimation theory and the Cramer-Rao bound to precision in quantum metrology
- Examples of applications
- Quantum sensing and metrology for more than one parameter and beyond the Cramer-Rao bound

Measurement and estimation

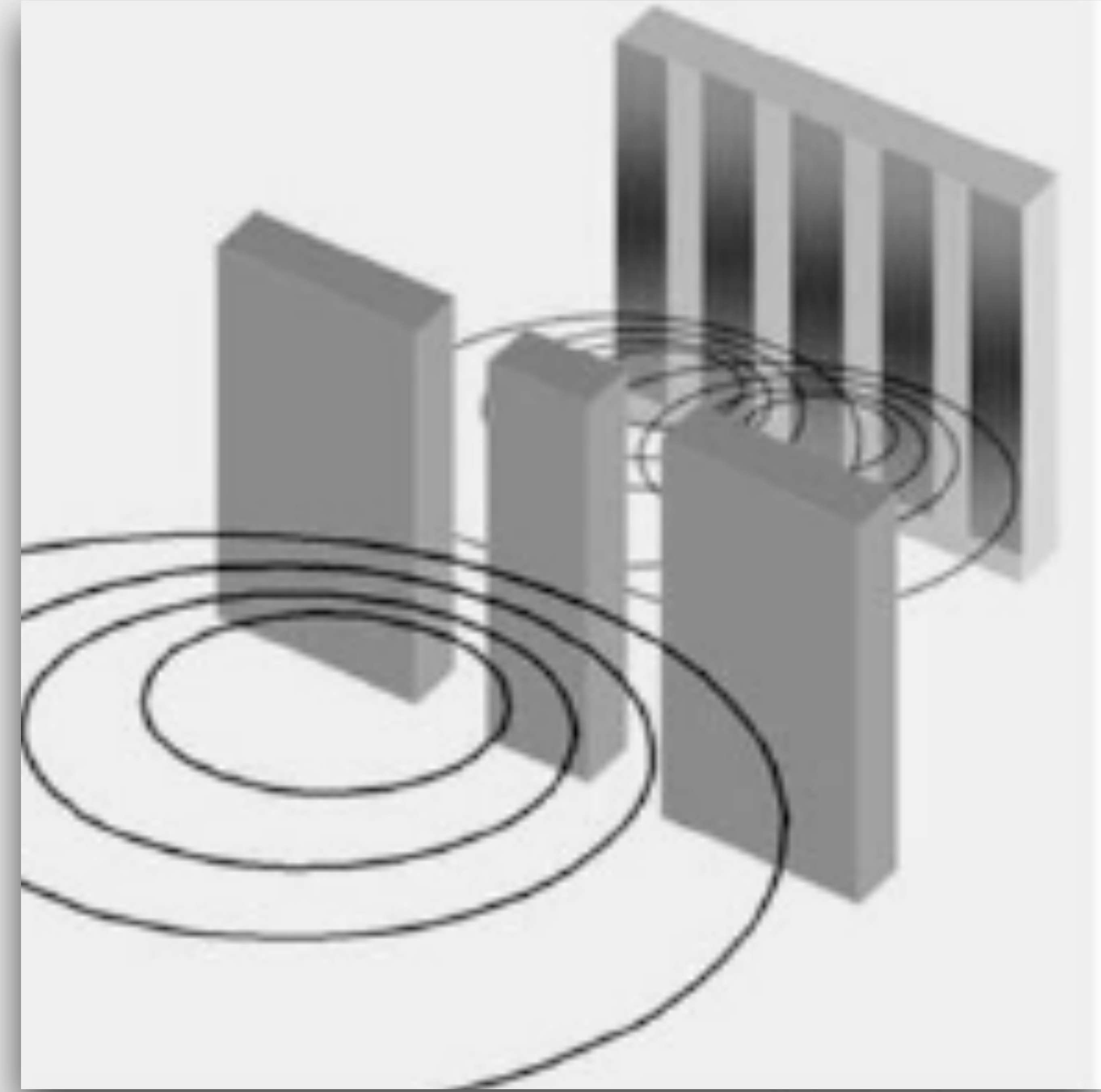
(tum.de)

Do we measure physical quantities?

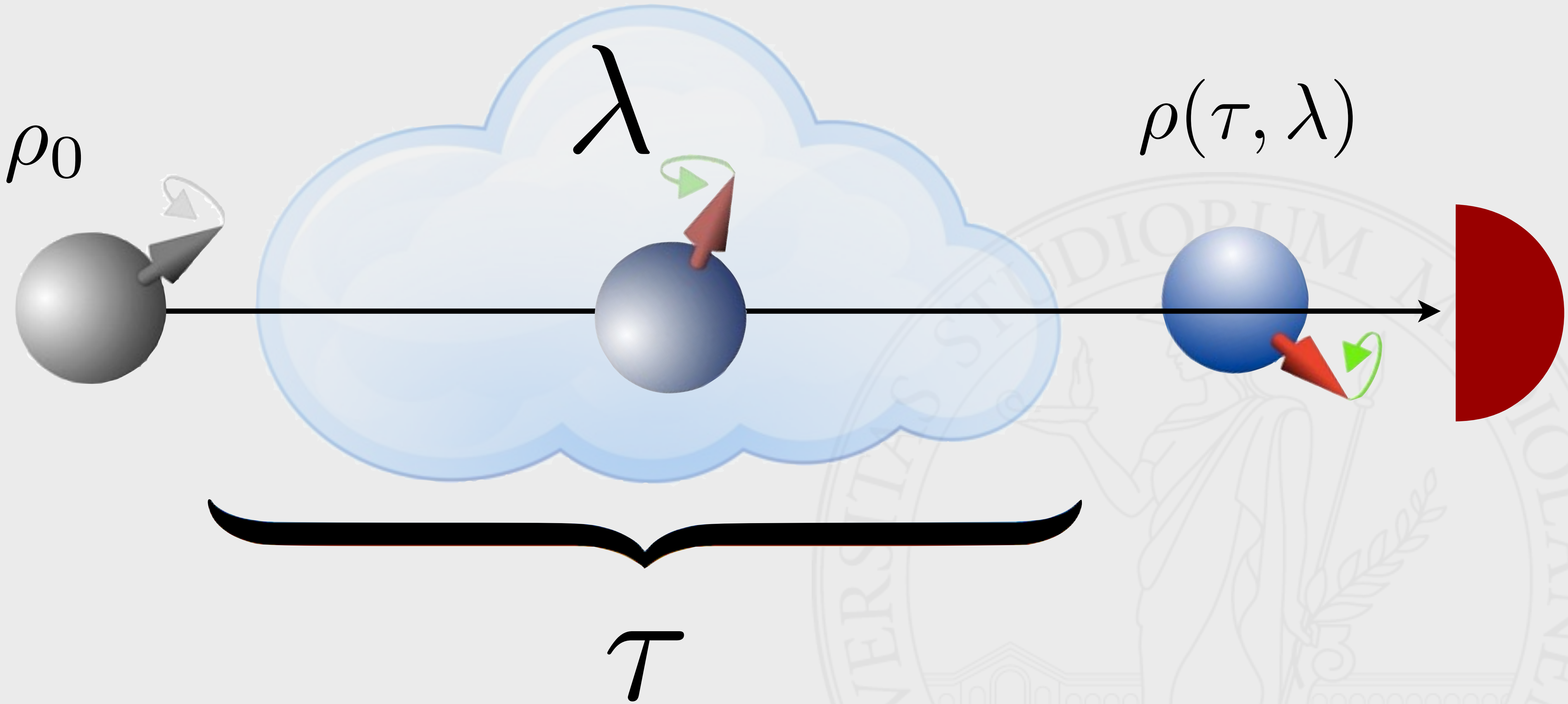
Measurement and estimation

Or perhaps we are mostly estimating them?

Measurement and estimation



Quantum Probing



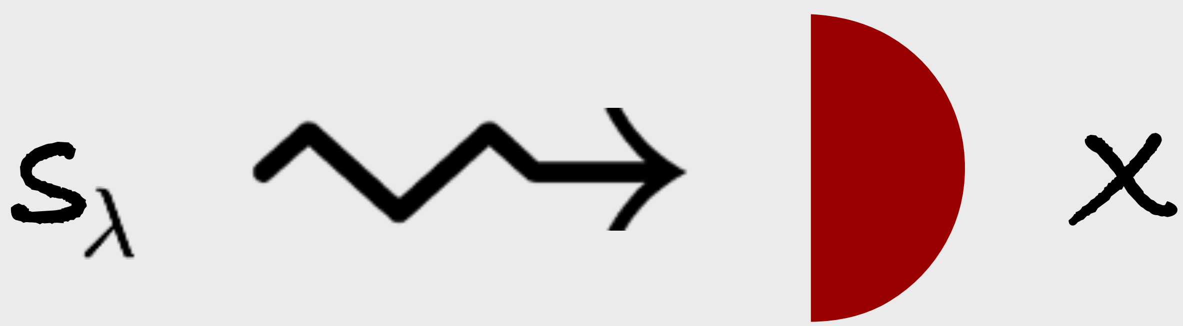
Estimation Theory

(forget the quantum for a while)

Measurement and estimation

- ~~direct measurements~~
- indirect measurements

→ influence on a different quantity



$$\boldsymbol{\chi} = (x_1, x_2, \ldots)$$

- choice of the measurement $\quad p(x|\lambda)$

- choice of the estimator $\quad \boldsymbol{\chi} \mapsto \hat{\lambda} = f(\boldsymbol{\chi})$

Measurement and estimation

- global estimation theory
  (when you have no a priori information)
  look for a measurement which is optimal in average
  (over the possible values of the parameter)

- local estimation theory
  (when you have some a priori information)
  look for a measurement which is optimal for a
  specific value of the parameter (—> ultimate bounds)

local estimation theory: Cramer - Rao bound

- variance of unbiased estimators

$$\mathrm{Var}_\lambda[\widehat{\lambda}] \geq \frac{1}{MF(\lambda)}$$

- M -> number of measurements
- F -> Fisher Information

$$F(\lambda) = \int dx\, p(x|\lambda) \Big[\partial_\lambda \log p(x|\lambda)\Big]^2$$

local estimation theory: Cramer - Rao bound

The proof of the Cramer-Rao bound is obtained by observing that given two functions $f_1(x)$ and $f_2(2)$ the average

$$\langle f_1, f_2\rangle = \int dx\, p(x|\lambda)\, f_1(x)\, f_2(x)$$

defines a scalar product. Upon chosing $f_1(x) = \hat{\lambda}(x) - \lambda$ and $f_2(x) = \partial_\lambda \ln p(x|\lambda)$ we have

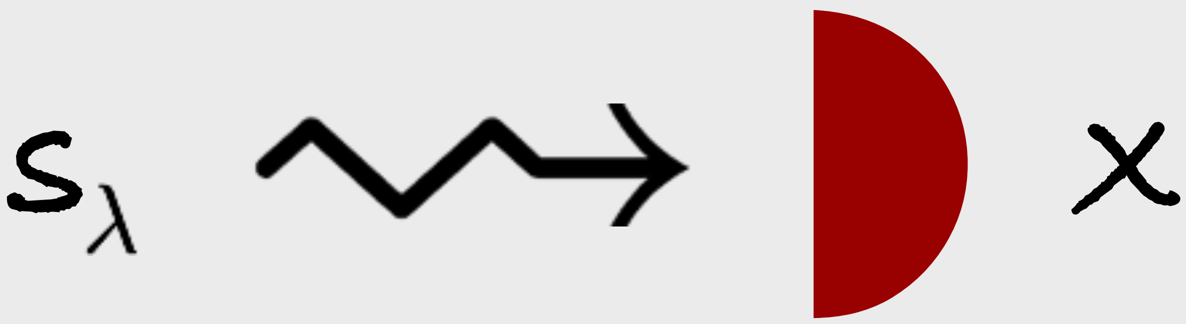
$$\begin{aligned} ||f_1||^2 &= \mathrm{Var}(\lambda) \\ ||f_2||^2 &= F(\lambda) \\ \langle f_1, f_2\rangle &= 1 \end{aligned}$$

$$\int dx\, \partial_\lambda p(x|\lambda) = 0$$

$x_1, x_2, ..., x_M$ independent we have $p(x_1, x_2, ..., x_M|\lambda) = \prod_{k=1}^M \log p(x_k|\lambda)$ and, in turn,

$$\begin{aligned} F_M(\lambda) &= \int dx_1...dx_M\, p(x_1, x_2, ..., x_M|\lambda)\, [\partial_\lambda \ln p(x_1, x_2, ..., x_M|\lambda)]^2 \\ &= M \int dx\, p(x|\lambda)\, [\partial_\lambda \ln p(x|\lambda)]^2 = M F(\lambda)\,. \end{aligned}$$

Optimal estimation scheme (classical)



$$\boldsymbol{\chi} = (x_1, x_2, \ldots)$$

$$\boldsymbol{\chi} \mapsto \hat{\lambda} = f(\boldsymbol{\chi})$$

$$\mathrm{Var}_\lambda[\hat{\lambda}] \geq \frac{1}{MF(\lambda)}$$

- Optimal measurement -> maximum Fisher (no recipes on how to find it)

- Optimal estimator -> saturation of CR inequality (e.g. Bayesian or MaxLik asymptotically)

Quantum Estimation Theory

Quantum estimation

- What about time and temperature in quantum mechanics?
- The "resources" involved in quantum-enahnced technology are entanglement, nonlocality, entropy, interferometric phase-shift, etc.. In general they are not observable quantities in strict sense (do not correspond to a selfadjoint operator)

  - No correspondence principle
  - No uncertainty relations

Quantum estimation

- What about time and temperature in quantum mechanics?
- The "resources" involved in quantum-enahnced technology are entanglement, nonlocality, entropy, interferometric phase-shift, etc.. In general they are not observable quantities in strict sense (do not correspond to a selfadjoint operator)

Quantum estimation theory

Quantum estimation

ϱ_λ

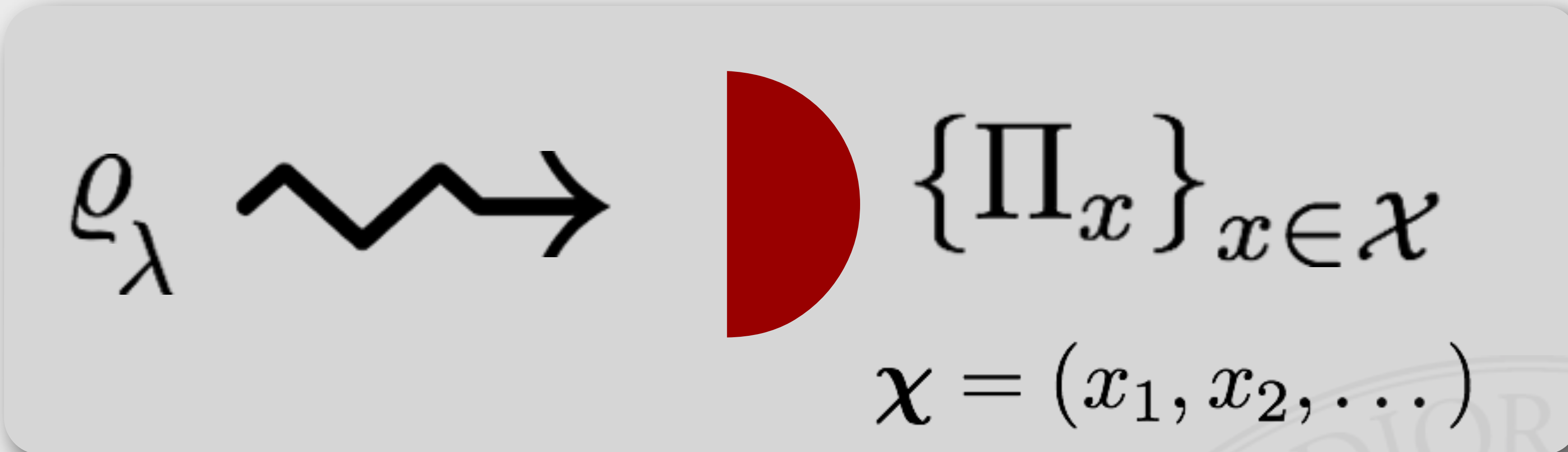
$\{\Pi_x\}_{x\in\mathcal{X}}$

$\chi = (x_1, x_2, \dots)$

- Optimal measurements
- Ultimate bounds to precision

UNIVERSITAS STUDIORUM MEDIOLANENSIS

Quantum estimation



- Probability density $\quad p(x|\lambda) = \mathrm{Tr}\left[\varrho_\lambda \, \Pi_x\right]$

UNIVERSITAS STUDIORUM MEDIOLANENSIS

Let's go quantum (local) (1)

ϱ_λ  $\{\Pi_x\}_{x\in\mathcal{X}}$

$\chi = (x_1, x_2, \dots)$

- probability density $p(x|\lambda) = \mathrm{Tr}\left[\varrho_\lambda \, \Pi_x\right]$

- symm. log. derivative (SLD) $$\frac{L_\lambda \varrho_\lambda + \varrho_\lambda L_\lambda}{2} = \frac{\partial \varrho_\lambda}{\partial \lambda}$$

  selfadjoint, zero mean $\mathrm{Tr}\left[\varrho_\lambda \, L_\lambda\right] = 0$

- Fisher information $$F(\lambda) = \int dx \, \frac{\mathrm{Re}\left(\mathrm{Tr}\left[\varrho_\lambda \Pi_x L_\lambda\right]\right)^2}{\mathrm{Tr}[\varrho_\lambda \Pi_x]}$$

Let's go quantum (local) (2)

$$F(\lambda) \leq \int dx \left| \frac{\mathrm{Tr}\left[\varrho_\lambda \Pi_x L_\lambda\right]}{\sqrt{\mathrm{Tr}[\varrho_\lambda \Pi_x]}} \right|^2$$

parameter independent POVM

$$= \int dx \left| \mathrm{Tr}\left[\frac{\sqrt{\varrho_\lambda}\sqrt{\Pi_x}}{\sqrt{\mathrm{Tr}\left[\varrho_\lambda \Pi_x\right]}} \sqrt{\Pi_x} L_\lambda \sqrt{\varrho_\lambda} \right] \right|^2$$

$$\leq \int dx \, \mathrm{Tr}\left[\Pi_x L_\lambda \varrho_\lambda L_\lambda\right]$$

$$= \mathrm{Tr}[L_\lambda \varrho_\lambda L_\lambda] = \mathrm{Tr}[\varrho_\lambda L_\lambda^2]$$

Helstrom 1976
Braunstein & Caves 1994

- Fisher vs Quantum Fisher

$$F(\lambda) \leq H(\lambda) \equiv \mathrm{Tr}[\varrho_\lambda L_\lambda^2] = \mathrm{Tr}[\partial_\lambda \varrho_\lambda L_\lambda]$$

- ultimate bound on precision $\mathrm{Var}(\lambda) \geq \dfrac{1}{M H(\lambda)}$

Optimal estimation scheme (quantum, local)

ϱ_λ

$$\{\Pi_x\}_{x\in\mathcal{X}}$$

$$\boldsymbol{\chi} = (x_1, x_2, \dots)$$

- Optimal measurement -> Fisher = quantum Fisher

  It is projective! The spectral measure of the SLD

- Optimal estimator -> saturation of CR inequality (classical postprocessing, e.g. Bayesian or MaxLix)

$$\boldsymbol{\chi} \mapsto \widehat{\lambda} = f(\boldsymbol{\chi})$$

General formulas (basis indepedent)

ϱ_λ 

$$\frac{L_\lambda \varrho_\lambda + \varrho_\lambda L_\lambda}{2} = \frac{\partial \varrho_\lambda}{\partial \lambda}$$

Lyapunov equation

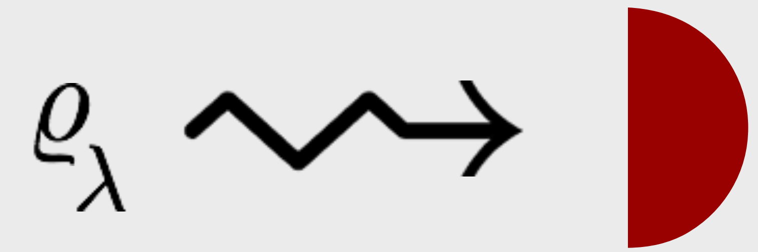
- Symmetric logarithmic derivative

$$L_\lambda = 2\int_0^\infty dt\, \exp\{-\varrho_\lambda t\}\, \partial_\lambda \varrho_\lambda\, \exp\{-\varrho_\lambda t\}$$

- Quantum Fisher Information

$$H(\lambda) = 2\int_0^\infty dt\, \mathrm{Tr}\left[\partial_\lambda \varrho_\lambda\, \exp\{-\varrho_\lambda t\}\, \partial_\lambda \varrho_\lambda\, \exp\{-\varrho_\lambda t\}\right]$$

General formulas



- Family of quantum states

$$\varrho_\lambda = \sum_n \varrho_n |\psi_n\rangle\langle\psi_n|$$

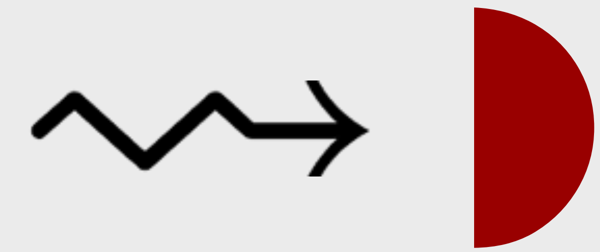
- Symmetric logarithmic derivative

$$L_\lambda = \sum_p \frac{\partial_\lambda \varrho_p}{\varrho_p} |\psi_p\rangle\langle\psi_p| + 2 \sum_{n\neq m} \frac{\varrho_n - \varrho_m}{\varrho_n + \varrho_m} \langle\psi_m|\partial_\lambda \psi_n\rangle \, |\psi_m\rangle\langle\psi_n|$$

- Quantum Fisher Information

$$H(\lambda) = \sum_p \frac{(\partial_\lambda \varrho_p)^2}{\varrho_p} + 2 \sum_{n\neq m} \frac{(\varrho_n - \varrho_m)^2}{\varrho_n + \varrho_m} |\langle\psi_m|\partial_\lambda \psi_n\rangle|^2$$

General formulas

ϱ_λ 

- **Family of quantum states**

$$\varrho_\lambda = \sum_n \varrho_n |\psi_n\rangle\langle\psi_n|$$

- **Symmetric logarithmic derivative**

$$L_\lambda = \sum_p \frac{\partial_\lambda \varrho_p}{\varrho_p} |\psi_p\rangle\langle\psi_p| + 2 \sum_{n\neq m} \frac{\varrho_n - \varrho_m}{\varrho_n + \varrho_m} \langle\psi_m|\partial_\lambda\psi_n\rangle \, |\psi_m\rangle\langle\psi_n|$$

- **Quantum Fisher Information**

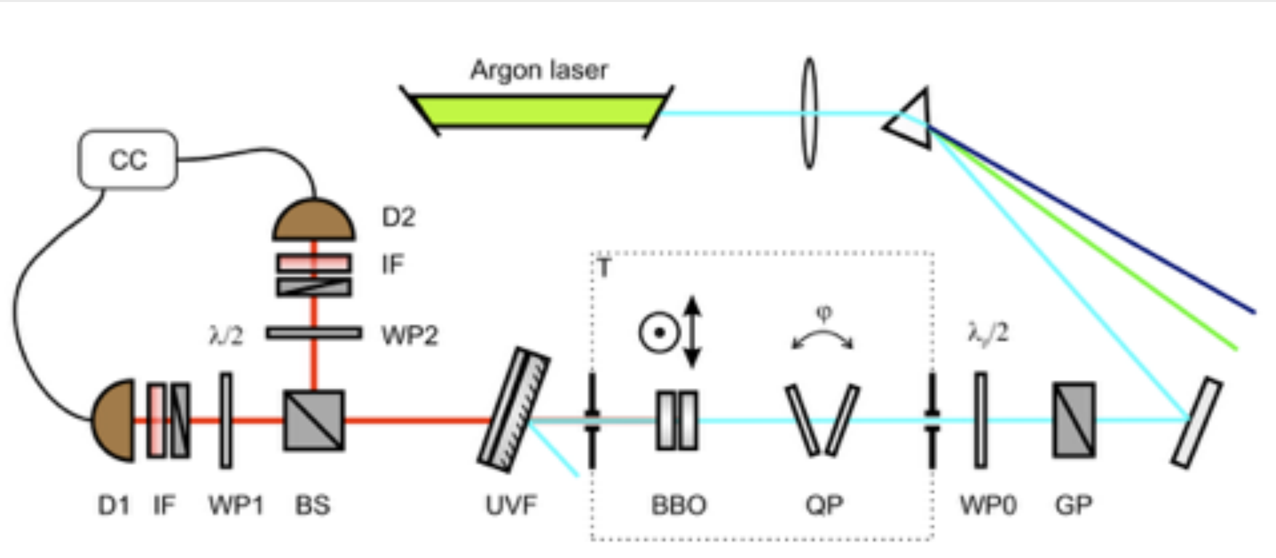
$$H(\lambda) = 8 \lim_{\epsilon\to 0} \frac{1 - F(\varrho_\lambda, \varrho_{\lambda+\epsilon})}{\epsilon^2}$$

Examples

Other applications (madamina il catalogo e' questo)

- Quantum Interferometry
- Estimation of Gaussian states and operations
- Coupling constants (e.g. nonlinear interactions)
- Wave function of finite-dimensional systems
- Estimation of entanglement (and discord)
- Estimation in quantum critical systems
- Assessing quantum probes for complex systems
- Assessing quantum resources in metrology
- Assessing local vs global measurements
- Assessing criticality as a resource in metrology
- Probing quantum phase transitions
- Probing Hamiltonian terms
- New physics at gravity/QM interface
- ...

Estimation of entanglement (@INRIM)



$$|\psi_\phi\rangle = \cos\phi|HH\rangle + \sin\phi|VV\rangle$$

$$D_\phi = \cos^2\phi|HH\rangle\langle HH| + \sin^2\phi|VV\rangle\langle VV|$$

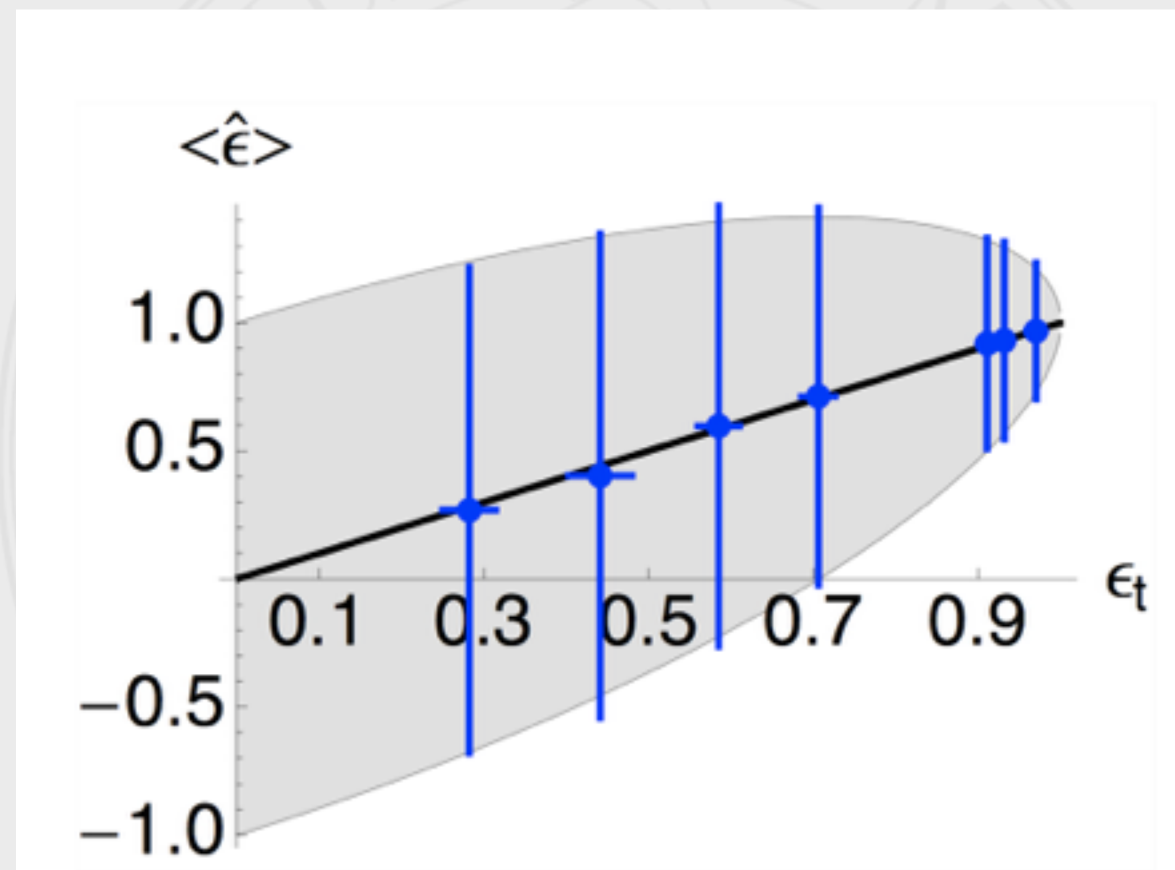
$$\varrho_\epsilon = p|\psi_\phi\rangle\langle\psi_\phi| + (1-p)D_\phi$$

$$\epsilon = p\sin 2\phi$$

optimal estimation by visibility measurements

Fisher information is monotone with entanglement

Estimation of entanglement is inherently inefficient



PRL 104, 100501 (2010) PHYSICAL REVIEW LETTERS week ending 12 MARCH 2010

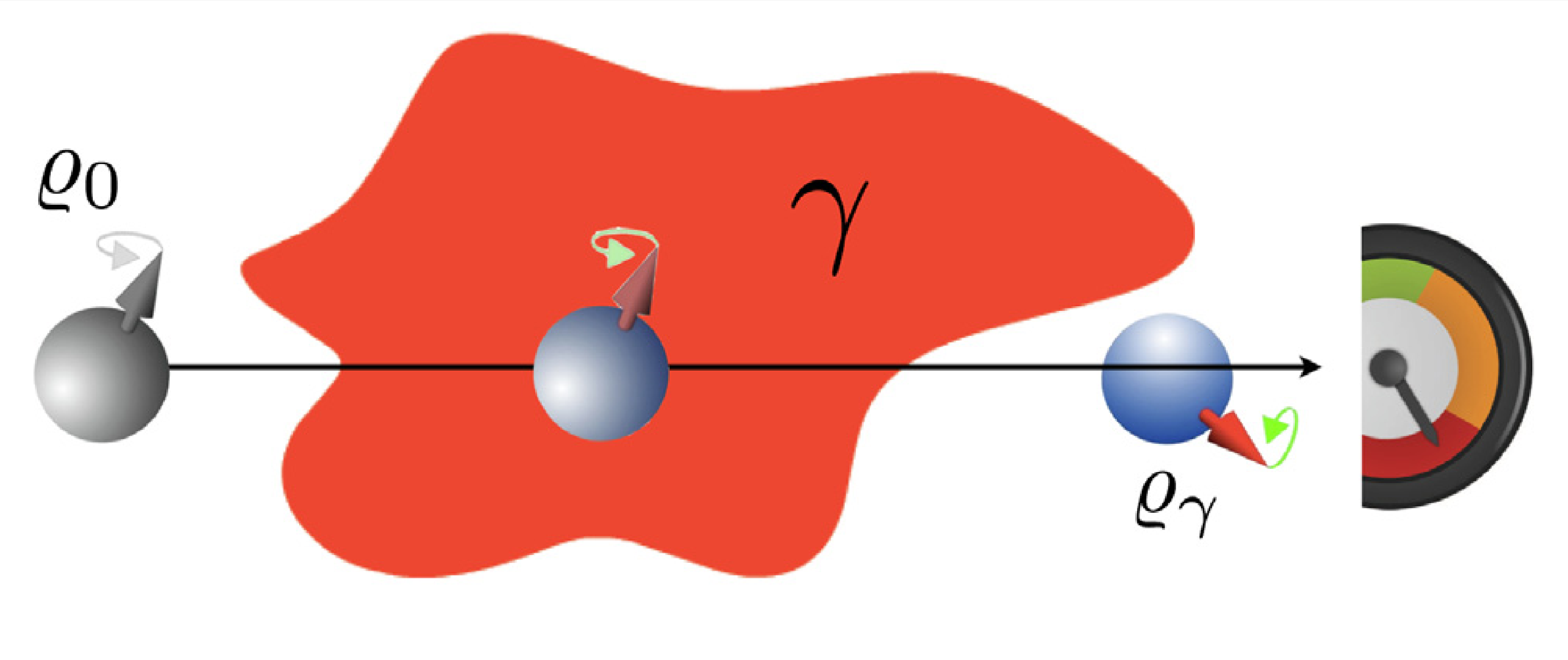
Experimental Estimation of Entanglement at the Quantum Limit

Giorgio Brida,[1] Ivo Pietro Degiovanni,[1] Angela Florio,[1,2] Marco Genovese,[1] Paolo Giorda,[3] Alice Meda,[1] Matteo G. A. Paris,[4,5] and Alexander Shurupov[6,1,7]

Quantum probes for complex systems

Quantum thermometry

Spectral characterisation



Quantum probes for complex systems

Quantum probes for the cutoff frequency of Ohmic environments

Claudia Benedetti,[1] Fahimeh Salari Sehdaran,[2] Mohammad H. Zandi,[2] and Matteo G. A. Paris[1]
[1]*Quantum Technology Lab, Physics Department, Università degli Studi di Milano, Milano, Italy*
[2]*Faculty of Physics, Shahid Bahonar University of Kerman, Kerman, Iran*

PRA **97**, 012126 (2018)

Quantum thermometry by single-qubit dephasing

Sholeh Razavian,[1] Claudia Benedetti,[2] Matteo Bina,[2] Yahya Akbari-Kourbolagh,[3] and Matteo G. A. Paris[2,4]
[1]*Faculty of Physics, Azarbaijan Shahid Madani University, Tabriz, Iran*
[2]*Quantum Technology Lab, Dipartimento di Fisica, Università degli Studi di Milano, I-20133 Milano, Italy*
[3]*Faculty of Physics, Azarbaijan Shahid Madani University, Tabriz, Iran*
[4]*INFN, Sezione di Milano, I-20133 Milano, Italy*

Eur. Phys. J. Plus **134,** 284 (2019)

Quantum metrology out of equilibrium

Sholeh Razavian[1,2], Matteo G. A. Paris[1,3]
[1]*QCSE School, Villa del Grumello, I-22100, Como, Italy*
[2]*Faculty of Physics, Azarbaijan Shahid Madani University, Tabriz, Iran*
[3]*Quantum Technology Lab, Dipartimento di Fisica 'Aldo Pontremoli', Università degli Studi di Milano, I-20133 Milano, Italy*

Physica A **525**, 825 (2019)

Universal Quantum Magnetometry with Spin States at Equilibrium

Filippo Troiani[1,*] and Matteo G. A. Paris[2,3,†]
[1]*Centro S3, CNR-Istituto di Nanoscienze, I-41125 Modena, Italy*
[2]*Quantum Technology Lab, Dipartimento di Fisica dell'Università degli Studi di Milano, I-20133 Milano, Italy*
[3]*INFN, Sezione di Milano, I-20133 Milano, Italy*

(Received 2 November 2017; published 29 June 2018)

PRL **120**, 260503 (2018)

Quantum probes for complex systems

Continuous-variable quantum probes for structured environments

Matteo Bina,[1,*] Federico Grasselli,[2,1] and Matteo G. A. Paris[1,3]
[1] *Quantum Technology Lab, Dipartimento di Fisica, Università degli Studi di Milano, I-20133 Milano, Italy*
[2] *Institut für Theoretische Physik III, Heinrich-Heine-Universität Düsseldorf, D-40225 Düsseldorf, Germany*
[3] *INFN, Sezione di Milano, I-20133 Milano, Italy*

PRA **97**, 012125 (2018)

The walker speaks its graph: global and nearly-local probing of the tunnelling amplitude in continuous-time quantum walks

J. Phys. A: Math. Theor. **52** (2019) 10530

Luigi Seveso, Claudia Benedetti and Matteo G A Paris
Quantum Technology Lab, Dipartimento di Fisica Aldo Pontremoli,
Università degli Studi di Milano, I-20133 Milano, Italy

The quantum walker probes her coin parameter

Shivani Singh* and C. M. Chandrashekar†
The Institute of Mathematical Sciences, C. I. T, campus, Taramani, Chennai, 600113, India and Homi Bhabha National Institute, Training School Complex, Anushakti Nagar, Mumbai 400094, India

Matteo G. A. Paris‡
Quantum Technology Lab, Dipartimento di Fisica Aldo Pontremoli,
Università degli Studi di Milano, I-20133 Milano, Italia

PRA **99**, 052117 (2019)

Quantum probes for universal gravity corrections

The existence of a minimum length and generalized uncertainty principle (GUP), influence all quantum Hamiltonians

Das et al PRL 101, 221301 (2008)

$$H = H_0 + H_1 + \mathcal{O}(\beta^2)$$

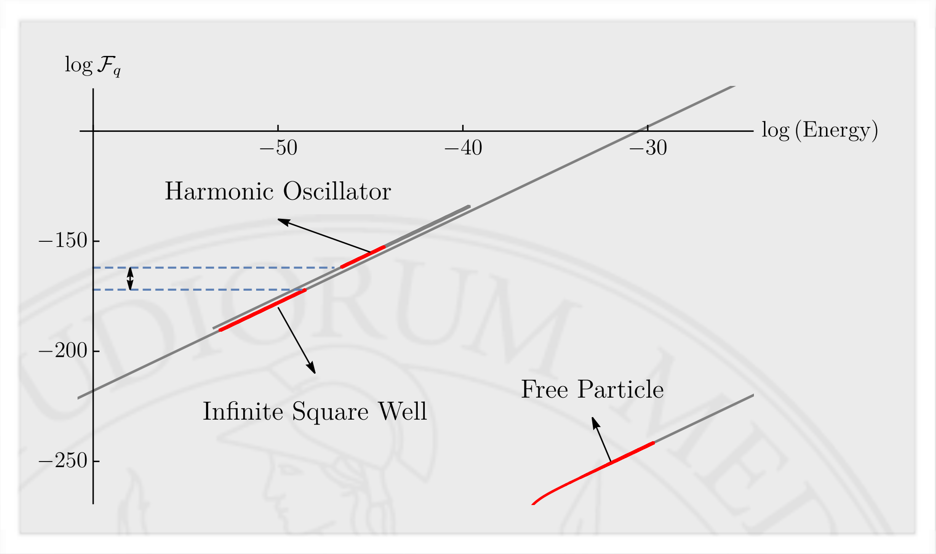
$$H_0 = \frac{p_0^2}{2m} + V(\vec{r}) \qquad H_1 = \frac{\beta}{m} p_0^4$$

$$\beta = \beta_0/(M_{\rm Pl}c)^2 = \ell_{\rm Pl}^2/2\hbar^2$$

Quantum probes for universal gravity corrections

the largest values of QFI are obtained with a quantum probe subject to a harmonic potential and initially prepared in a superposition of perturbed energy eigenstates

QFI is superadditive with the dimenssion of the system, which therefore represents a metrological resource. The gain in precision is not due to the appearance of entanglement of the state but rather to the increasing number of superposed states generated by the perturbation.



PHYSICAL REVIEW D **102**, 056012 (2020)

Quantum probes for universal gravity corrections

Alessandro Candeloro,[1,*] Cristian Degli Esposti Boschi,[2] and Matteo G. A. Paris[1,3,†]

[1]*Quantum Technology Lab, Dipartimento di Fisica Aldo Pontremoli, Università degli Studi di Milano, I—20133 Milano, Italy*
[2]*CNR-IMM, Sezione di Bologna, Via Gobetti 101, I—40129 Bologna, Italy*
[3]*INFN-Sezione di Milano, I-20133 Milano, Italy*

More than one parameter

The multiparametric case

$$\boldsymbol{X} = \{x_1, x_2, \ldots\}$$

$$\Sigma_{jk} = E\left[\left(\hat{\lambda}_j - E[\hat{\lambda}_j]\right)\left(\hat{\lambda}_k - E[\hat{\lambda}_k]\right)\right]$$

$$E[f] = \int d\mathbf{x}\; p(\mathbf{x}|\boldsymbol{\lambda})\, f(\mathbf{x}) \qquad \mathbf{x} \to \text{data}$$

Symmetric and non-symmetric LD

$$\partial_k \rho_{\underline{\lambda}} = \frac{1}{2}\left[L_k \rho_{\underline{\lambda}} + \rho_{\underline{\lambda}} L_k\right]$$

L_k self adjoint

R_k not necessarily selfadjoint

$$\partial_k \rho_{\underline{\lambda}} = \rho_{\underline{\lambda}} R_k \qquad \partial_k \equiv \frac{\partial}{\partial \lambda_k}$$

$$H_{\mu\nu}(\underline{\lambda}) = \mathrm{Tr}\left[\rho_{\underline{\lambda}} \frac{1}{2}\left(L_\mu L_\nu + L_\nu L_\mu\right)\right]$$

$$J_{\mu\nu}(\underline{\lambda}) = \mathrm{Tr}\left[\rho_{\underline{\lambda}} R_\mu R_\nu^{\dagger}\right]$$

Symmetric and non-symmetric bounds

$$\mathrm{Tr}[\Sigma W] \geq C_S(\lambda, W) \qquad C_S(\lambda, W) = \mathrm{Tr}[H^{-1}(\lambda) W]$$

$$\mathrm{Tr}[\Sigma W] \geq C_R(\lambda, W)$$

$$C_R(\lambda, W) = \mathrm{Tr}[\mathrm{Re}\, J^{-1} W] + \| \sqrt{W}\, \mathrm{Im}\, J^{-1} \sqrt{W} \|_1$$

$$\mathrm{Tr}[\Sigma W] \geq \max(C_R, C_S)$$

Neither the SLD nor the RLD bound are in general achievable.

The SLD could not be achievable because it corresponds to the bound obtained by measuring optimally and simultaneously each single parameter, and this is not possible when the optimal measurements do not commute.

The RLD bound could not be achievable because the optimal estimator does not always correspond to a proper quantum measurement (that is, a proper positive operator valued measure).

The Holevo bound and how not to deal with it

A scalar bound known as Holevo bound has been independently obtained and proved tighter than both C_R and C_S i.e.

$$C_H(\lambda, W) \geq \max(C_R, C_S)$$

$$C_S(\lambda, W) \leq C_H(\lambda, W) \leq (1+R)\, C_S(\lambda, W)$$

where $R = \| i\, H^{-1}(\lambda)\, D(\lambda) \|_\infty$ $\quad$ $0 \leq R \leq 1$

$$D_{\mu\nu} = -\frac{i}{2} \mathrm{Tr}\left[\rho_\lambda [L_\mu, L_\nu] \right]$$

($[L_\mu, L_\nu]$ → commutator)

and $\|A\|_\infty$ denotes the largest eigenvalue of A

Beyond the Cramer-Rao bound

Back to premises $p(x|\lambda)$

The classical CR holds under the assumption:

- The sample space is independent on the parameter to be estimated

To obtain QCR we added another assumption:

- The measurement POVM is independent on the parameter to be estimated

Parameter-dependent measurements: no Cramer-Rao

$$
\begin{aligned}
F(\lambda) &\leq \int dx \left| \frac{\mathrm{Tr}\left[\varrho_\lambda \Pi_x L_\lambda\right]}{\sqrt{\mathrm{Tr}[\varrho_\lambda \Pi_x]}} \right|^2 \\
&= \int dx \left| \mathrm{Tr}\left[\frac{\sqrt{\varrho_\lambda}\sqrt{\Pi_x}}{\sqrt{\mathrm{Tr}\left[\varrho_\lambda \Pi_x\right]}} \sqrt{\Pi_x} L_\lambda \sqrt{\varrho_\lambda} \right] \right|^2 \\
&\leq \int dx \, \mathrm{Tr}\left[\Pi_x L_\lambda \varrho_\lambda L_\lambda\right] \\
&= \mathrm{Tr}[L_\lambda \varrho_\lambda L_\lambda] = \mathrm{Tr}[\varrho_\lambda L_\lambda^2]
\end{aligned}
$$

Parameter-dependent measurements

$$\int dx\, m_\lambda(x)\, \Pi_\lambda(x) = \mathbb{I}$$

Parameter-dependent sample space
(possible also in classical
estimation problem)

Parameter-dependent POVM
(an entirely novel quantum
degree of freedom)

New bound for parameter dependent POVMs

- gravimetry with a quantum mechanical oscillator

$$\mathcal{H} = p^2/2m + kx^2/2 + mgx$$

- prepare the oscillator in a coherent state

$$H(g) = 8m/\omega^3 \sin^2 \omega t/2$$

$$F_{\mathcal{H}}(g) = 2m/\omega^3$$ measurement of energy (Hamiltonian)

L Seveso, MAC Rossi, MGA Paris, Phys. Rev. A **95**, 012111 (2017)

New bound for parameter dependent POVMs

$$F_X(\lambda) = \int d\nu \left\{ \frac{[\mathrm{tr}(\Pi_\lambda(x)\partial_\lambda\rho_\lambda)]^2}{\mathrm{tr}(\Pi_\lambda(x)\rho_\lambda)} + \frac{[\mathrm{tr}(\partial_\lambda\Pi_\lambda(x)\rho_\lambda)]^2}{\mathrm{tr}(\Pi_\lambda(x)\rho_\lambda)} + \frac{2\,\mathrm{tr}(\Pi_\lambda(x)\partial_\lambda\rho_\lambda)\,\mathrm{tr}(\partial_\lambda\Pi_\lambda(x)\rho_\lambda)}{\mathrm{tr}(\Pi_\lambda(x)\rho_\lambda)} \right\}$$

$$F_X(\lambda) \leq \left[\sqrt{H(\lambda)} + \sqrt{\mathscr{K}_X(\lambda)}\right]^2$$

(projective POVMs)

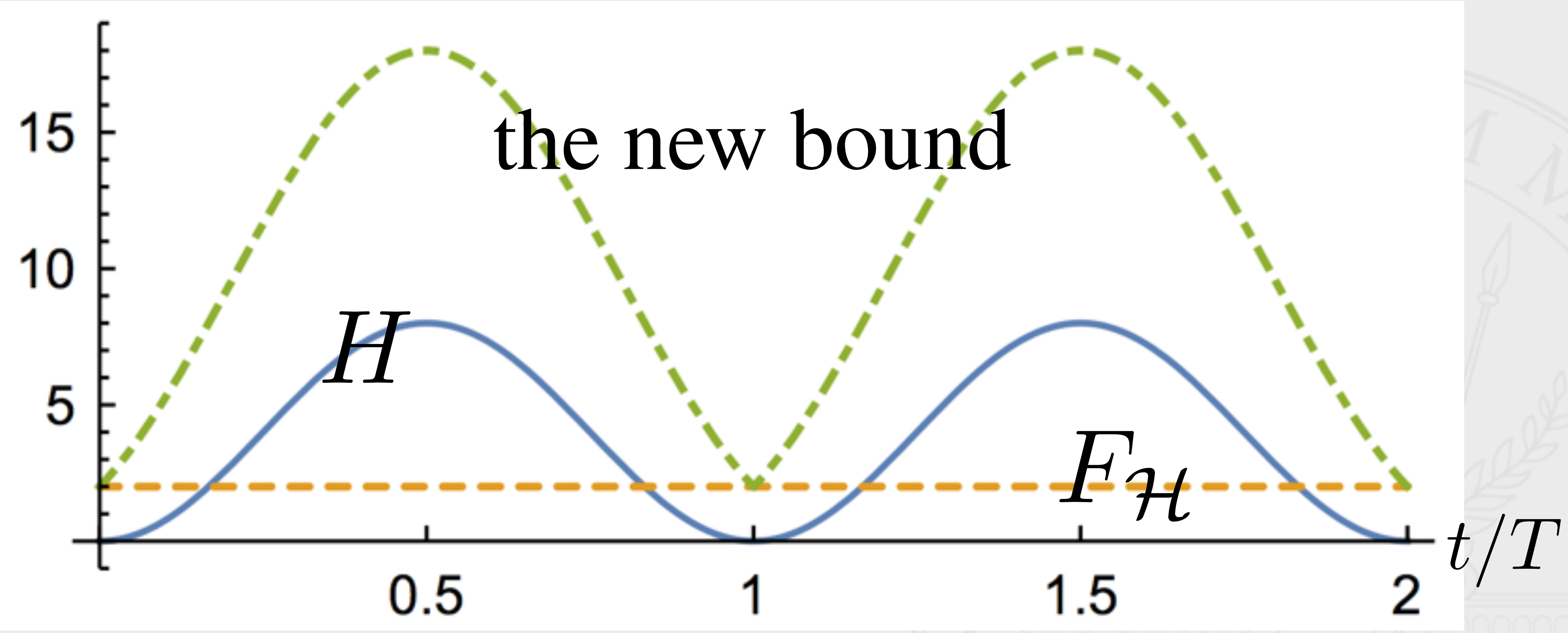
$$\mathscr{K}_X(\lambda) = 4\int dx\, \langle \partial_\lambda x|\rho_\lambda|\partial_\lambda x\rangle$$

Achievable ?

What about optimal measurement?

L Seveso, MAC Rossi, MGA Paris, Phys. Rev. A **95**, 012111 (2017)

New bound for parameter dependent POVMs



Summary/conclusions

Quantum estimation theory is a relevant tool to design and assess quantum enhanced measurements (estimation schemes)

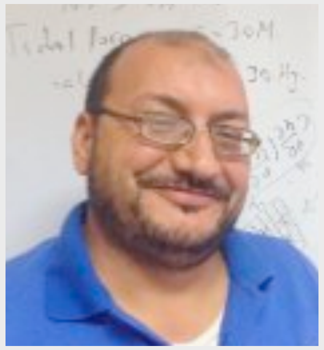
The single parameter QCR provides the ultimate quantum limit to precision. More precisely, it bounds precision of schemes exploiting quantumness of probes.

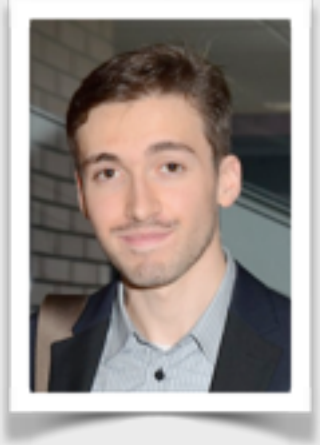
Quantum-based measurements may be further improved by exploiting detector dependence on the parameter of interest and thus the quantumness of detectors, i.e. quantum-enhanced measurements may be more precise than previously thought.

Current research is about joint estimation of more than one parameter, e.g. signal and noise to realize self calibrating estimation schemes.

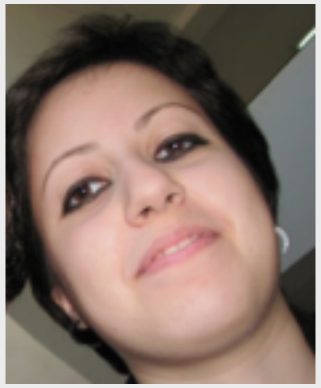


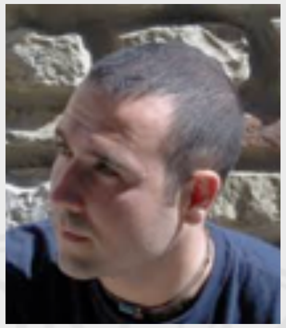
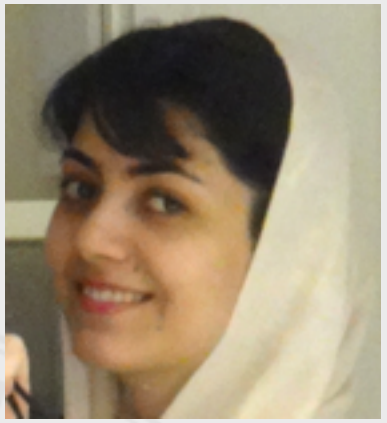








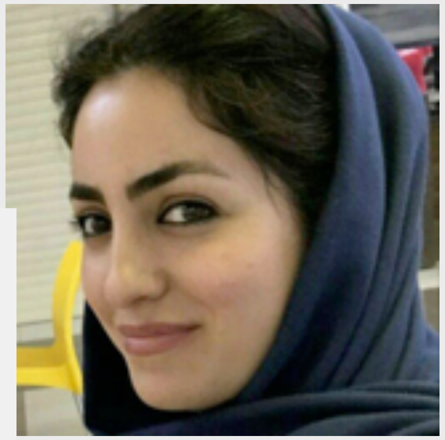



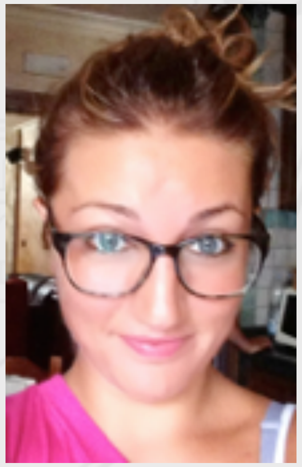
could not find a picture of Hamza

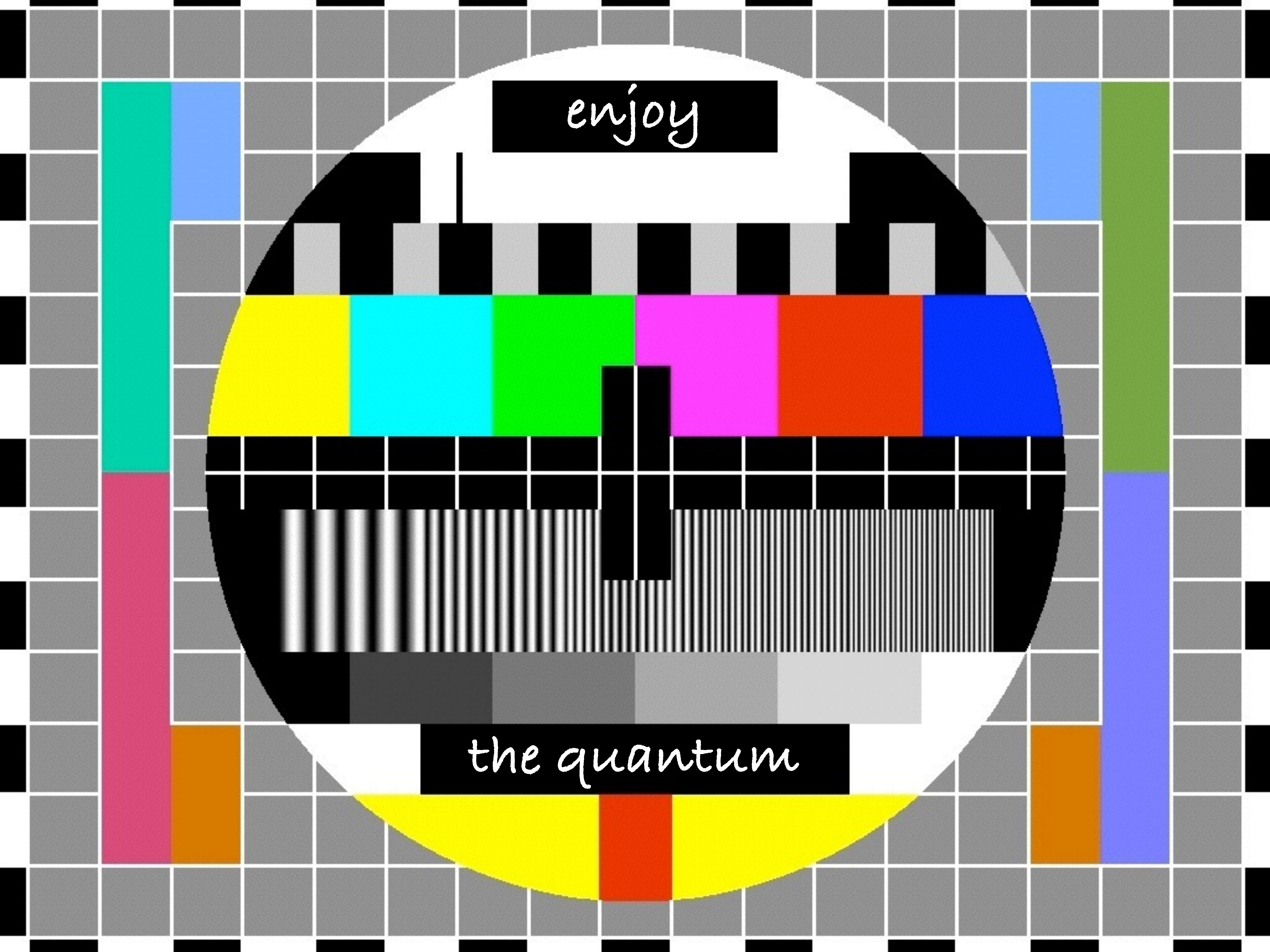









enjoy
the quantum