IICQI International Iran Conferences on Quantum Information

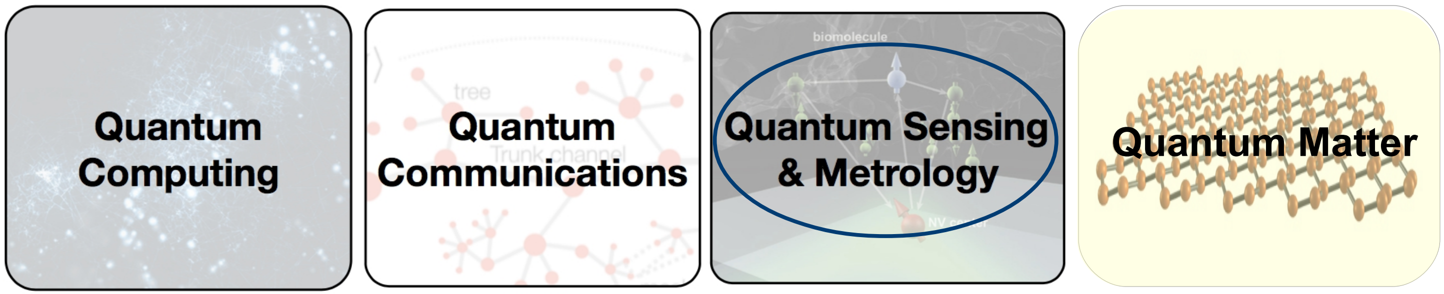
Qubit sensors in correlated noise environments:
Advances in noisy quantum metrology

Thu • 4 Mar 2021

Lorenza Viola
lorenza.viola@dartmouth.edu
Dept. Physics & Astronomy
Dartmouth

NSF
Fonds de recherche Nature et technologies Québec
EQUS
Quantum Information Science at Dartmouth
DARTMOUTH

Broad context: Quantum frontiers



- Quantum science offers unique, unprecedented capabilities for fundamental and practical advances toward 'taming quantum complexity' – by supplying means for:
  - ➞ Performing computational and simulation tasks that are intractable classically...
  - ➞ Transmitting information in ways that are intrinsically secure...
  - ➞ Pushing quantum measurements to their ultimate precision limits...
  - ➞ Designing and probing new states of matter, at equilibrium and beyond...

- Determining the extent to which 'quantum advantages' may be achievable *in practice* demands unprecedented level of understanding and control over noise effects...
  - ➞ '<u>Noisy</u> Intermediate-Scale Quantum' ⇒ 'large' size of target quantum system, N > 50 qubits...

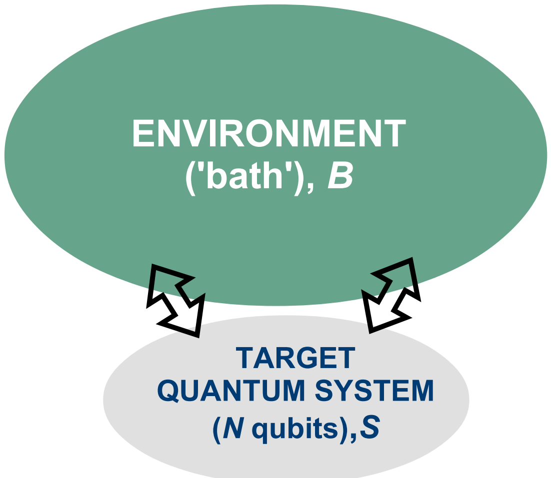
Preskill, *Quantum* **2**, 79 (2018).

<u>Challenge</u>: To obtain accurate, predictive characterization of open quantum dynamics – in natural and engineered quantum systems – under *realistic noise environments*...

Quantum frontiers (a theorist's view...)

- Theoretical and computational approaches to [controlled] open quantum system dynamics necessarily involve one or more *simplifying assumptions*, including:
  - Initial system-bath factorization
  - Initial thermal equilibrium ⇒ Stationary noise
  - Weak (linear) system-bath coupling ⇒ Perturbative treatments
  - Gaussianity ⇒ Gaussian noise statistics
  - Classicality ⇒ Stochastic ('commuting') noise
  - Lack of temporal correlations (white noise, 'Markovianity')
  - Lack of spatial correlations ('independent noise')

  ⋮



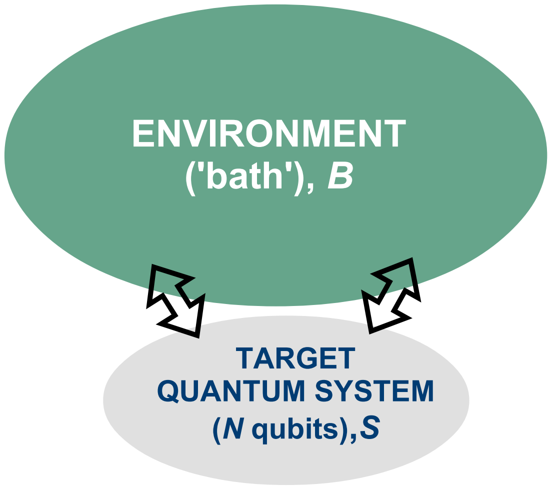
- On the one end: Each of these assumptions requires careful scrutiny – they cannot be taken as *a priori* valid nor do they need to be quantitatively accurate for tasks of interest...
- On the other end: Increasing complexity of NISQ-scale devices and tasks prevents complete theoretical descriptions (or brute-force numerics) from remaining viable...

Challenge (again): To achieve successful *model reduction* – open-system characterization, control analysis, and synthesis must strike a balance between simplicity and accuracy...

Quantum frontiers (a theorist's view...)

- Theoretical and computational approaches to [controlled] open quantum system dynamics necessarily involve one or more *simplifying assumptions*, including:
  - Initial system-bath factorization
  - Initial thermal equilibrium ⇒ Stationary noise
  - Weak (linear) system-bath coupling ⇒ Perturbative treatments
  - Gaussianity ⇒ Gaussian noise statistics
  - Classicality ⇒ Stochastic ('commuting') noise
  - Lack of temporal correlations (white noise, 'Markovianity')
  - Lack of spatial correlations ('independent noise')

  ⋮



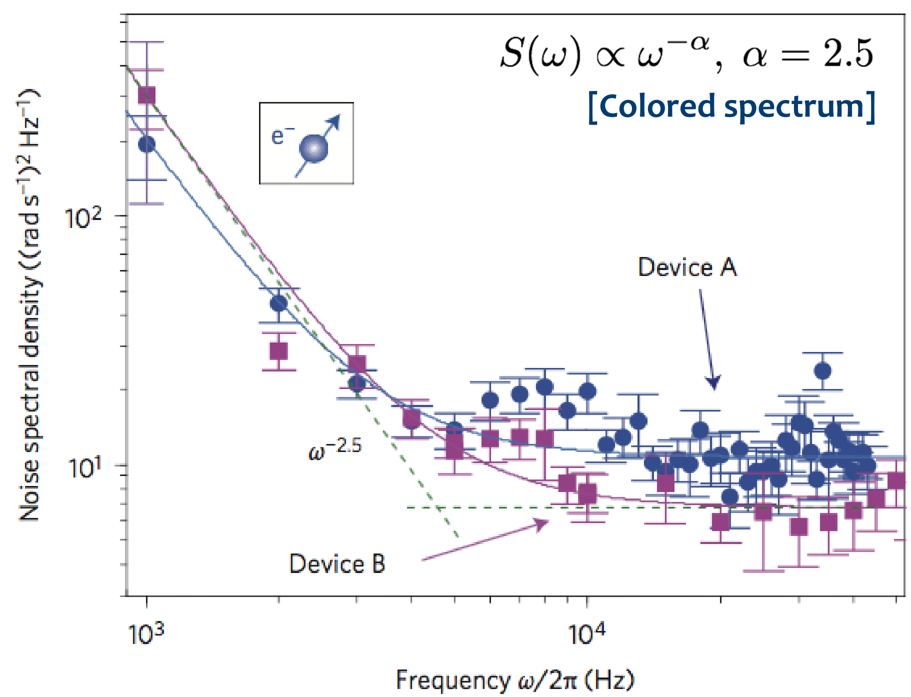
- On the one end: Each of these assumptions requires careful scrutiny – they cannot be taken as *a priori* valid nor do they need to be quantitatively accurate for tasks of interest...
- On the other end: Increasing complexity of NISQ-scale devices and tasks prevents complete theoretical descriptions (or brute-force numerics) from remaining viable...

Challenge (again): To achieve successful *model reduction* – open-system characterization, control analysis, and synthesis must strike a balance between simplicity and accuracy...

Focus: Spatiotemporally correlated noise

- Many [most?] realistic noise sources generically exhibit non-trivial spatiotemporal correlations:
  - ➞ The occurrence of *temporal correlations* has been verified across a variety of systems through *dynamical decoupling* and *quantum noise spectroscopy* experiments…

Bylander *et al*, Nat. Phys. **7** (2011); Muhonen *et al*, Nat. Nanotech. **9** (2014); Romach *et al*, PRL **114** (2015); Malinowski *et al*, *ibid.* **118** (2017)…



Focus: Spatiotemporally correlated noise

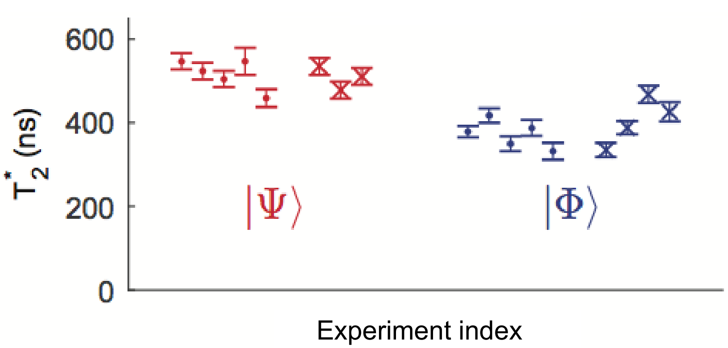
- Many [most?] realistic noise sources generically exhibit non-trivial spatiotemporal correlations:
  - ➞ The occurrence of *temporal correlations* has been verified across a variety of systems through *dynamical decoupling* and *quantum noise spectroscopy* experiments...

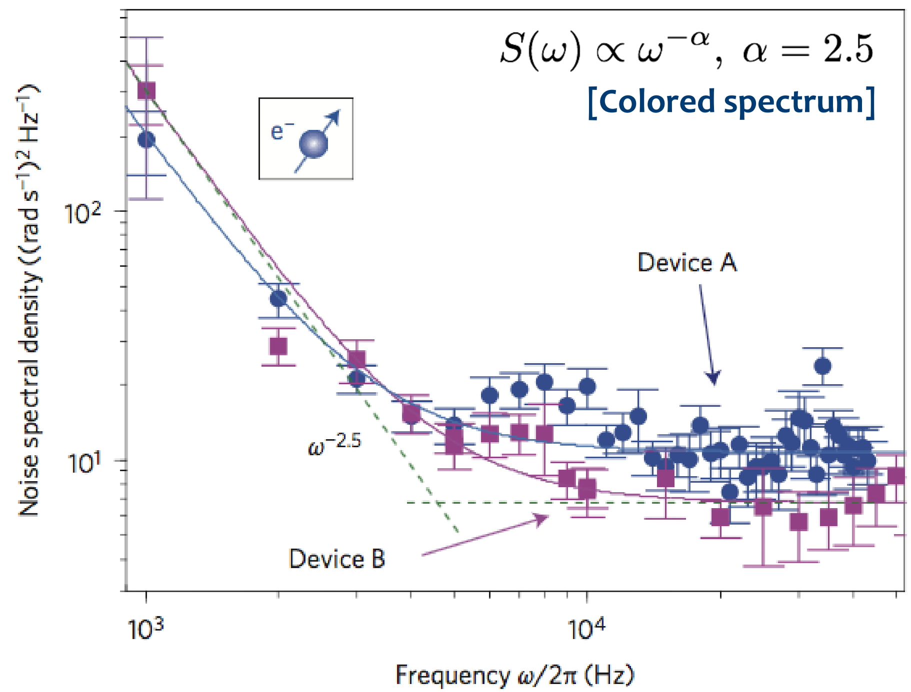
    Bylander *et al*, Nat. Phys. **7** (2011); Muhonen *et al*, Nat. Nanotech. **9** (2014); Romach *et al*, PRL **114** (2015); Malinowski *et al*, *ibid.* **118** (2017)...

  - ➞ *Spatially correlated* low-frequency noise has been reported for Si/SiGe two-qubit devices...

    Boter *et al*, PRB **101** (2020).

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|\downarrow\uparrow\rangle - i|\uparrow\downarrow\rangle) \qquad |\Phi\rangle = \frac{1}{\sqrt{2}}(|\downarrow\downarrow\rangle - i|\uparrow\uparrow\rangle)$$





Focus: Spatiotemporally correlated quantum noise

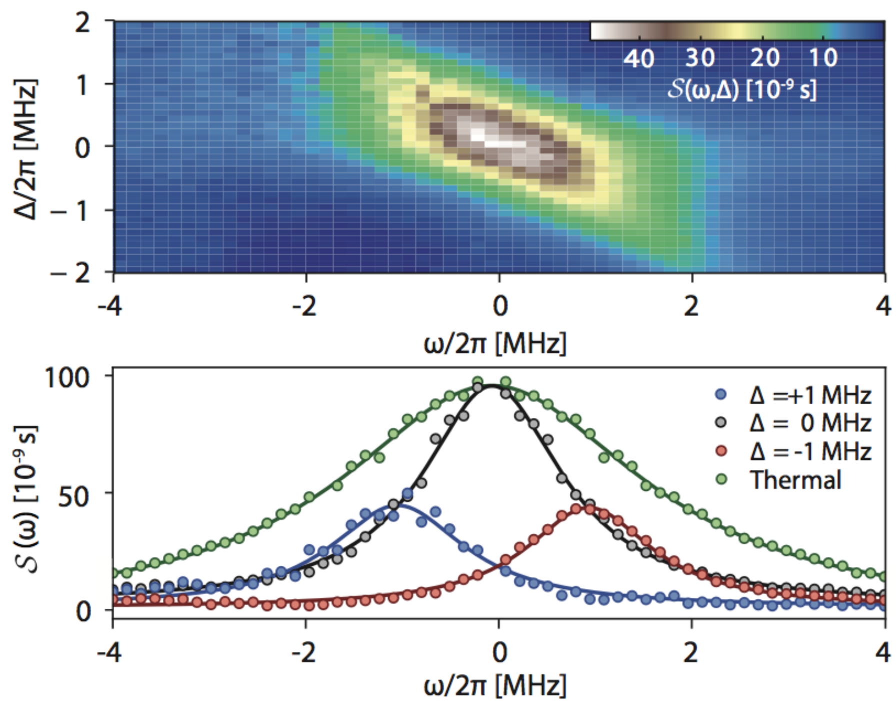
- Many [most?] realistic noise sources generically exhibit non-trivial spatiotemporal correlations and may entail *intrinsically non-commuting* degrees of freedom:

$$S(\omega) = \int_{-\infty}^{\infty} dt\, e^{-i\omega t} \langle B(t)B(0)\rangle_{\mathrm{B}} = \frac{1}{2}[S^{+}(\omega) + S^{-}(\omega)] \neq S(-\omega)$$

Classical spectrum (symmetric) **Quantum spectrum (asymmetric)**

➞ *Non-classical* noise environments have been directly probed in recent experiments, for both single- and two-qubit devices...

Quintana *et al*, PRL **118** (2017);
Yan *et al*, *ibid.* **120** (2018)...



$$S(\omega) = e^{\beta\hbar\omega} S(-\omega)$$

[Fluctuation-dissipation at equilibrium]

Focus: Spatiotemporally correlated quantum noise

- Many [most?] realistic noise sources generically exhibit non-trivial spatiotemporal correlations and may entail *intrinsically non-commuting* degrees of freedom:

$$S(\omega) = \int_{-\infty}^{\infty} dt\, e^{-i\omega t} \langle B(t)B(0)\rangle_{\mathrm{B}} = \frac{1}{2}[S^{+}(\omega) + S^{-}(\omega)] \neq S(-\omega)$$

Classical spectrum (symmetric) | Quantum spectrum (asymmetric)

→ *Non-classical* noise environments have been directly probed in recent experiments, for both single- and two-qubit devices...

Quintana *et al*, PRL **118** (2017);
Yan *et al*, *ibid.* **120** (2018)...

PRX QUANTUM **1,** 010305 (2020)

Two-Qubit Spectroscopy of Spatiotemporally Correlated Quantum Noise in Superconducting Qubits

Uwe von Lüpke,[1,†] Félix Beaudoin,[2,3] Leigh M. Norris,[2] Youngkyu Sung,[1] Roni Winik,[1] Jack Y. Qiu,[1] Morten Kjaergaard,[1] David Kim,[4] Jonilyn Yoder,[4] Simon Gustavsson,[1] Lorenza Viola,[2] and William D. Oliver[1,4,5,*]

[Also featured in Phys. Rev. Journal Club, https://journals.aps.org/journal-club]

Motivation and outline

- <u>Key task</u>: Quantum metrology – to achieve precision measurements by employing *quantum sensors and distinctively quantum effects*:
  - ➞ Entanglement and squeezing;
  - ➞ Quantum correlations beyond entanglement, indistinguishability;
  - ➞ Dynamical non-linearities, interaction…

Pezzè *et al*, RMP **90**, 035005 (2018);
Braun *et al*, *ibid*. 035006 (2018).

(Q1) How does correlated quantum noise impact *entanglement-assisted parameter estimation*?
(Q2) What *experimental metrological settings* may be directly/most affected?
(Q3) How/how well can effects of correlated quantum noise be *effectively countered*?

PHYSICAL REVIEW A **98**, 020102(R) (2018)

Rapid Communications

Ramsey interferometry in correlated quantum noise environments

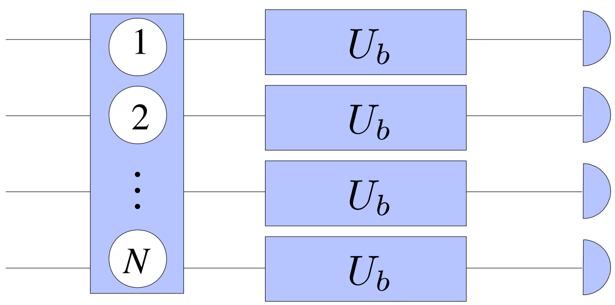
Félix Beaudoin, Leigh M. Norris, and Lorenza Viola
Department of Physics and Astronomy, Dartmouth College, 6127 Wilder Laboratory, Hanover, New Hampshire 03755, USA

Francisco Riberi

Riberi, Norris, Beaudoin & LV, forthcoming (2021).

Noiseless quantum metrology

- Goal: To exploit entanglement between *N* qubit sensors to achieve *super-classical precision scaling* in parameter estimation, for specified resources – fixed N and T (or, N and ν).



Preparation **Evolution, t** **Measurement, estimator** $\hat{b}(\vec{\mu})$

$$\nu = T/t$$
$$\vec{\mu} \equiv \{\mu_1, \ldots, \mu_\nu\}$$

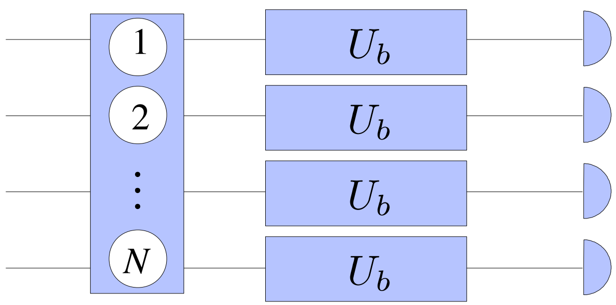
$$\langle \hat{b}(\vec{\mu}) \rangle_{\vec{\mu}} = b$$ **Unbiasedness**

$$\Delta^2 \hat{b} \equiv \sum_{\vec{\mu}} P(\vec{\mu}|b)(\hat{b}(\vec{\mu}) - b)^2$$

$$\Delta \hat{b} \equiv \sqrt{\Delta^2 \hat{b}}$$ **Precision**

Noiseless quantum metrology

❖ <u>Goal</u>: To exploit entanglement between *N* qubit sensors to achieve *super-classical precision scaling* in parameter estimation, for specified resources – fixed N and T (or, N and ν).



Preparation **Evolution, t** **Measurement, estimator** $\hat{b}(\vec{\mu})$

$$\nu = T/t$$
$$\vec{\mu} \equiv \{\mu_1, \ldots, \mu_\nu\}$$

$$\langle \hat{b}(\vec{\mu}) \rangle_{\vec{\mu}} = b \qquad \textbf{Unbiasedness}$$

$$\Delta^2 \hat{b} \equiv \textstyle\sum_{\vec{\mu}} P(\vec{\mu}|b)(\hat{b}(\vec{\mu}) - b)^2$$

$$\Delta \hat{b} \equiv \sqrt{\Delta^2 \hat{b}} \qquad \textbf{Precision}$$

❖ The variance of any unbiased estimator is *lower-bounded by the Cramér-Rao bound*,

$$\Delta^2 \hat{b} \geq \Delta^2 \hat{b}_{\rm CR} = \frac{1}{\nu F_{\rm cl}[P(\vec{\mu}|b)]}, \quad F_{\rm cl}[P(\vec{\mu}|b)] \equiv \sum_{\vec{\mu}} \frac{1}{P(\vec{\mu}|b)} \left(\frac{\partial P(\vec{\mu}|b)}{\partial b} \right)^2$$

Classical Fisher information

➞ Maximizing classical FI over all possible POVMs yields *quantum FI*, $F_{\rm Q}[\rho_b] \equiv \max_{\{\mathcal{E}\}} F_{\rm cl}[\rho_b]$.

❖ The ultimate precision limit for noiseless estimation is set by the *quantum Cramér-Rao bound*,

$$\Delta^2 \hat{b}_{\rm CR} \geq \Delta^2 \hat{b}_{\rm QCR} = \frac{1}{\nu F_{\rm Q}[\rho_b]}$$

Entanglement-assisted noiseless quantum metrology

❖ The QFI of *any initial separable state* of *N* qubits is upper-bounded by *N* ⇒ the ultimate classically achievable precision is independent of measurement and estimator used:

$$\Delta\hat{b}_{\mathrm{SQL}} = \frac{1}{t\sqrt{N\nu}} = \frac{1}{\sqrt{NtT}}$$

Standard Quantum Limit

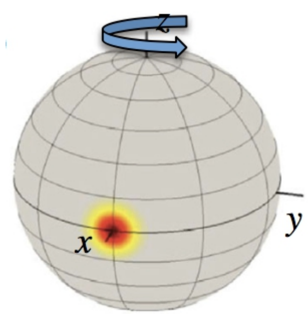
➡ *Super-classical precision scaling* requires that $F_{\mathrm{Q}}[\rho_b] > N$ ...

❖ The QFI attains its maximum value if the initial state exhibits genuine *N-partite entanglement* ⇒ the *ultimate precision limit is improved by a factor of* $N^{-1/2}$ *for same, fixed resources*:

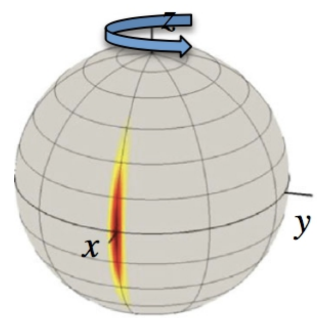
$$\Delta\hat{b}_{\mathrm{HL}} = \frac{1}{Nt\sqrt{\nu}} = \frac{1}{N\sqrt{tT}}$$

Heisenberg Limit

Coherent Spin State



Spin-Squeezed State



CSSs are *optimal separable states*,
$\Delta\hat{b} = \Delta\hat{b}_{\mathrm{SQL}} \propto N^{-1/2}$

SSSs possess *metrologically useful entanglement*,
$\Delta\hat{b} \propto N^{-1}$

Wineland *et al*, PRA **46** (1992); Kitagawa & Ueda, *ibid*. **47** (1993).

Entanglement-assisted noisy quantum metrology

- The impact of noise on predicted metrological advantages has been extensively studied...
  - ➞ No super-classical scaling permitted under fully uncorrelated [Markovian, independent] noise

    Huelga *et al*, PRL **79** (1997); Escher *et al*, Nat. Phys. **7** (2011)...

  - ➞ The presence of noise correlations has generally [so far?] been found to be less adversarial:
    - *Temporally correlated, independent noise*: DD cannot restore Heisenberg scaling; but, super-classical precision may be achieved at short detection times ('Zeno-like' regime).

      Chin *et al*, PRL **109** (2012)... Sekatski et al, NJP **18** (2016)...

    - *Spatially correlated, white noise*: Super-classical scaling achievable by DFS encodings; or, noise effects may be filtered from signal by tailored QEC...

      Dorner, NJP **14** (2012); Layden & Cappellaro, npj QI **4** (2018)...

    - *Spatially and temporally correlated, classical noise*: Memory effects may be beneficial to retain enhanced sensitivity over longer times...

      Szankowski, Trippenbach & Chwedenczuk, PRA **90** (2014).

Entanglement-assisted noisy quantum metrology

❖ The impact of noise on predicted metrological advantages has been extensively studied...

➞ No super-classical scaling permitted under fully uncorrelated [Markovian, independent] noise

Huelga *et al*, PRL **79** (1997); Escher *et al*, Nat. Phys. **7** (2011)...

➞ The presence of noise correlations has generally [so far?] been found to be less adversarial:

- *Temporally correlated, independent noise*: DD cannot restore Heisenberg scaling; but, super-classical precision may be achieved at short detection times ('Zeno-like' regime).

  Chin *et al*, PRL **109** (2012)... Sekatski et al, NJP **18** (2016)...

- *Spatially correlated, white noise*: Super-classical scaling achievable by DFS encodings; or, noise effects may be filtered from signal by tailored QEC...

  Dorner, NJP **14** (2012); Layden & Cappellaro, npj QI **4** (2018)...

- *Spatially and temporally correlated, classical noise*: Memory effects may be beneficial to retain enhanced sensitivity over longer times...

  Szankowski, Trippenbach & Chwedenczuk, PRA **90** (2014).

❖ Qubit sensors coupled to a *common, quantum noise environment* can become entangled in an uncontrolled way, which opens the door to an additional source of uncertainty...

➞ Especially relevant to spin-squeezing generation by coupling to bosonic modes.

Bohnet *et al*, Science **352** (2016); Hu et al, PRA **96** (2017)...

Noisy Ramsey interferometry: Setting

- Task: Frequency estimation by *N* qubit sensors under *correlated quantum dephasing* noise,

$$H_{SB}(t) = \frac{\hbar}{2}\sum_{n=1}^{N}\left[y_0(t)b + y(t)B_n(t)\right]\sigma_n^z$$

b = Target frequency parameter

→ Allow for the possibility of open-loop control via $y_0(t), y(t)$ (= 1 if no control is applied)

- Noise spectra acquire distinctive features relative to classical setting:

$$S_{nm}(\omega) = \int_{-\infty}^{\infty} \mathrm{d}t\, e^{-i\omega t}\langle B_n(t)B_m(0)\rangle_{\mathrm{B}} = \frac{1}{2}\left[S^{+}_{nm}(\omega) + S^{-}_{nm}(\omega)\right]$$

Classical spectra **Quantum spectra**

$$S^{+}_{nm}(\omega) = \int_{-\infty}^{\infty} \mathrm{d}t\, e^{-i\omega t}\langle [B_n(t)B_m(0)]_{+}\rangle_{\mathrm{B}} \qquad S^{-}_{nm}(\omega) = \int_{-\infty}^{\infty} \mathrm{d}t\, e^{-i\omega t}\langle [B_n(t)B_m(0)]_{-}\rangle_{\mathrm{B}}$$

→ Quantum spectra vanish for classical noise. Quantum non-commutativity manifests in *different symmetry properties* that spectra obey,

$$[S^{+}_{nm}(\omega)]^{*} = S^{+}_{nm}(-\omega), \quad [S^{-}_{nm}(\omega)]^{*} = -S^{-}_{nm}(-\omega) \quad \Rightarrow \quad S_{nm}(\omega) \neq S_{nm}(-\omega)$$

Noisy Ramsey interferometry: Setting

- Task: Frequency estimation by N qubit sensors under *correlated quantum dephasing* noise,

$$H_{SB}(t) = \frac{\hbar}{2}\sum_{n=1}^{N}[y_0(t)b + y(t)B_n(t)]\,\sigma_n^z$$

b = Target frequency parameter

➞ Allow for the possibility of open-loop control via $y_0(t), y(t)$ (= 1 if no control is applied)

- Noise spectra acquire distinctive features relative to classical setting:

$$S_{nm}(\omega) = \int_{-\infty}^{\infty} \mathrm{d}t\, e^{-i\omega t}\langle B_n(t)B_m(0)\rangle_{\mathrm{B}} = \frac{1}{2}\left[S^+_{nm}(\omega) + S^-_{nm}(\omega)\right]$$

Classical spectra (S^+_{nm}) **Quantum spectra** (S^-_{nm})

$$S^+_{nm}(\omega) = \int_{-\infty}^{\infty} \mathrm{d}t\, e^{-i\omega t}\langle [B_n(t)B_m(0)]_+\rangle_{\mathrm{B}} \qquad S^-_{nm}(\omega) = \int_{-\infty}^{\infty} \mathrm{d}t\, e^{-i\omega t}\langle [B_n(t)B_m(0)]_-\rangle_{\mathrm{B}}$$

➞ Quantum spectra vanish for classical noise. Quantum non-commutativity manifests in *different symmetry properties* that spectra obey,

$$[S^+_{nm}(\omega)]^* = S^+_{nm}(-\omega), \quad [S^-_{nm}(\omega)]^* = -S^-_{nm}(-\omega) \quad \Rightarrow \quad S_{nm}(\omega) \neq S_{nm}(-\omega)$$

- Noise spectra encode information about *spatiotemporal noise correlations*

$$C_{nm}(t) \equiv \langle B_n(t)B_m(0)\rangle_{\mathrm{B}}$$

→ $c_{nm}\delta(t)$ Temporally uncorrelated (white)

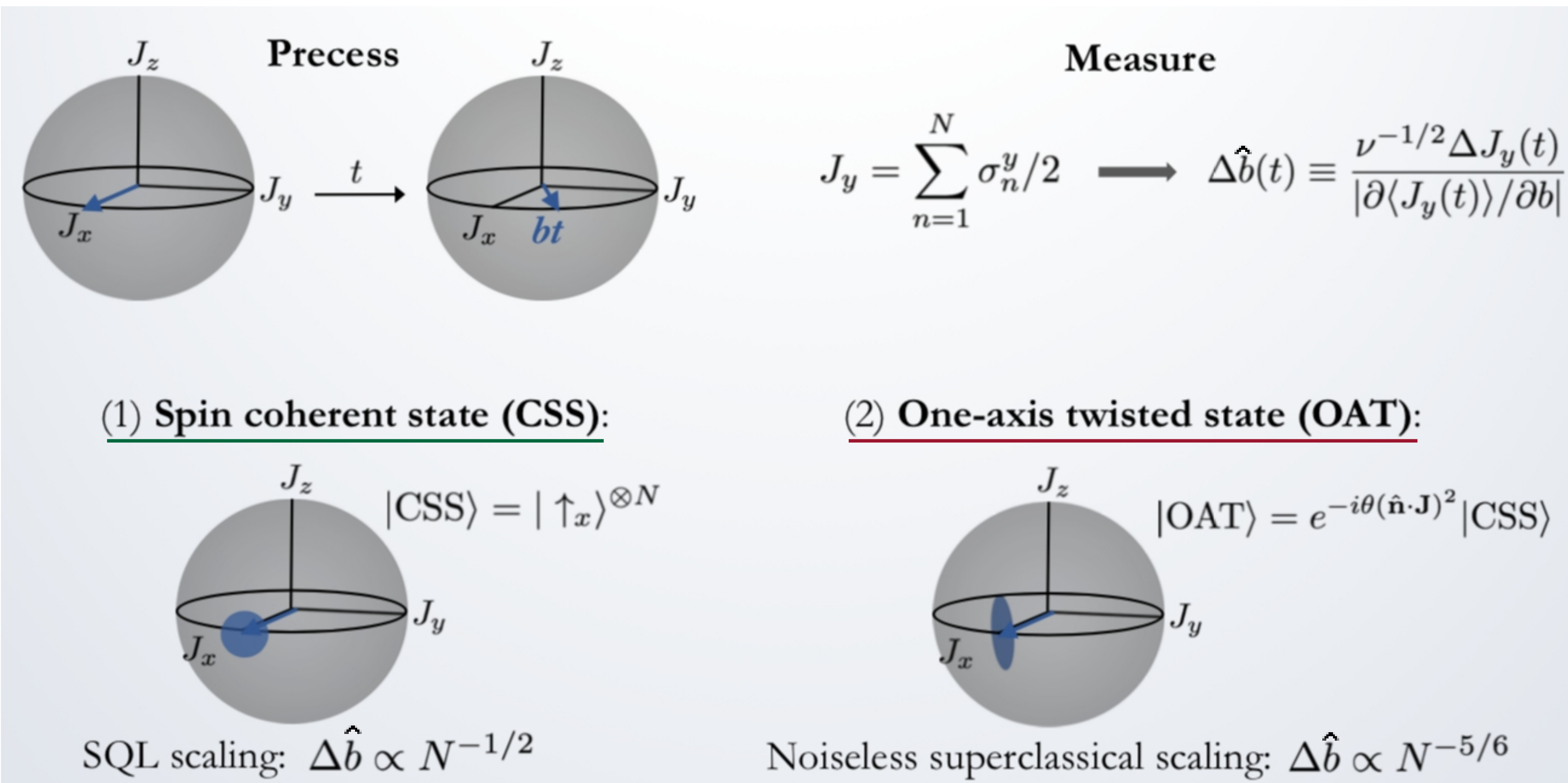
→ $\delta_{nm} f_n(t)$ Spatially uncorrelated

Noisy Ramsey interferometry: Protocol

❖ Task: Frequency estimation by *N* qubit sensors under *quantum correlated dephasing* noise

$$H_{SB}(t) = \frac{\hbar}{2}\sum_{n=1}^{N}\left[y_0(t)b + y(t)B_n(t)\right]\sigma_n^z \qquad S_{nm}(\omega) = \int_{-\infty}^{\infty} \mathrm{d}t\, e^{-i\omega t}\langle B_n(t)B_m(0)\rangle_{\mathrm{B}}$$

➞ Allow for the possibility of open-loop control via $y_0(t), y(t)$ (= 1 if no control is applied)



Noisy Ramsey interferometry: Reduced dynamics

❖ Quantities needed to determine the estimation precision:

$$\langle J_y \rangle = \sum_{n=1}^{N} \langle \sigma_n^y \rangle /2 \qquad \langle J_y^2 \rangle = N/4 + \sum_{n\neq m} \langle \sigma_n^y \sigma_m^y \rangle /4$$

➞ Exact results may be obtained through a cumulant expansion over bath operators:

$$\langle \sigma_n^y \rangle = e^{-\chi_{nn}(t)/2}\, \mathrm{Tr}_S \left[e^{-i\Phi_n(t)} \rho_S(0)\, \sigma_n^y \right]$$

$$\langle \sigma_n^y \sigma_m^y \rangle = e^{-[\chi_{nn}(t)+\chi_{mm}(t)]/2}\, \mathrm{Tr}_S \left[e^{-i\Phi_{nm}(t)} \rho_S(0)\, \sigma_n^y \sigma_m^y \right]$$

in terms of 'decay parameters' $\chi_{nm}(t)$ and 'effective propagators' $\Phi_n(t), \Phi_{nm}(t)$.

❖ Contributions mediated by classical spectra:

$$\chi_{nm}(t) = \frac{1}{2\pi} \mathrm{Re} \int_0^\infty d\omega\, F^+(\omega,t)\, S_{nm}^+(\omega)$$

Decay of coherences

Control filter function (FF) $F^+(\omega,t) \equiv \left| \int_0^t \mathrm{d}s\, y(s) e^{-i\omega s} \right|^2 = \frac{4\sin^2(\omega t/2)}{\omega^2}$ (no control)

Noisy Ramsey interferometry: Reduced dynamics

- Quantities needed to determine the estimation precision:

$$\langle J_y\rangle = \sum_{n=1}^{N}\langle\sigma_n^y\rangle/2 \qquad \langle J_y^2\rangle = N/4 + \sum_{n\neq m}\langle\sigma_n^y\sigma_m^y\rangle/4$$

➞ Exact results may be obtained through a cumulant expansion over bath operators:

$$\langle\sigma_n^y\rangle = e^{-\chi_{nn}(t)/2}\,\mathrm{Tr}_S\left[e^{-i\Phi_n(t)}\rho_S(0)\,\sigma_n^y\right]$$

$$\langle\sigma_n^y\sigma_m^y\rangle = e^{-[\chi_{nn}(t)+\chi_{mm}(t)]/2}\,\mathrm{Tr}_S\left[e^{-i\Phi_{nm}(t)}\rho_S(0)\,\sigma_n^y\sigma_m^y\right]$$

in terms of 'decay parameters' $\chi_{nm}(t)$ and 'effective propagators' $\Phi_n(t), \Phi_{nm}(t)$.

- Contributions mediated by quantum spectra:

$$\Psi_{nm}(t) = \frac{1}{2\pi}\mathrm{Im}\int_0^\infty d\omega\, F^-(\omega,t)\, S_{nm}^-(\omega)$$

Phase contributions

Control FF $\quad F^-(\omega,t) \equiv \int_0^t \mathrm{d}s\, y(s)\int_0^s \mathrm{d}u\, y(u)\, e^{-i\omega(u-s)}$

$$\Phi_n(t) = \varphi(t)\sigma_n^z + \sum_{\ell,\ell\neq n}\Psi_{n\ell}(t)\sigma_n^z\sigma_\ell^z$$

$$\varphi(t) \equiv b\int_0^t \mathrm{d}s\, y_0(s), \quad \Phi_{nm}(t) = \varphi(t)(\sigma_n^z+\sigma_m^z) - i\chi_{nm}(t)\sigma_n^z\sigma_m^z + \sum_{\ell,\ell\neq nm}\left[\Psi_{n\ell}(t)\sigma_n^z\sigma_\ell^z + \Psi_{m\ell}(t)\sigma_m^z\sigma_\ell^z\right]$$

Noisy Ramsey interferometry: Reduced dynamics

❖ Quantities needed to determine the estimation precision:

$$\langle J_y \rangle = \sum_{n=1}^{N} \langle \sigma_n^y \rangle /2 \qquad \langle J_y^2 \rangle = N/4 + \sum_{n\neq m} \langle \sigma_n^y \sigma_m^y \rangle /4$$

➞ Exact results may be obtained through a cumulant expansion over bath operators:

$$\langle \sigma_n^y \rangle = e^{-\chi_{nn}(t)/2}\, \underbrace{\mathrm{Tr}_S\left[e^{-i\Phi_n(t)} \rho_S(0)\, \sigma_n^y \right]}$$

$$\langle \sigma_n^y \sigma_m^y \rangle = e^{-[\chi_{nn}(t)+\chi_{mm}(t)]/2}\, \underbrace{\mathrm{Tr}_S\left[e^{-i\Phi_{nm}(t)} \rho_S(0)\, \sigma_n^y \sigma_m^y \right]}$$

(1) Initial CSS: Traces can be evaluated *exactly*

(2) Initial OAT: Traces can be evaluated through a *cumulant expansion over qubit operators* (truncated to 2nd-order)

❖ Simplest [but most adversarial?] setting: *Collective*, permutation-invariant noise regime

$$B_n(t) \equiv B(t),\ \forall n, t \quad \Rightarrow \quad \chi_{nm}(t) \equiv \chi(t),\ \Psi_{nm}(t) \equiv \Psi(t)$$

➞ For collective noise, non-zero phase parameter $\Psi(t)$ is distinctive of *non-classical* dephasing noise that is *both* spatially and temporally correlated.

Results: Spin-boson model

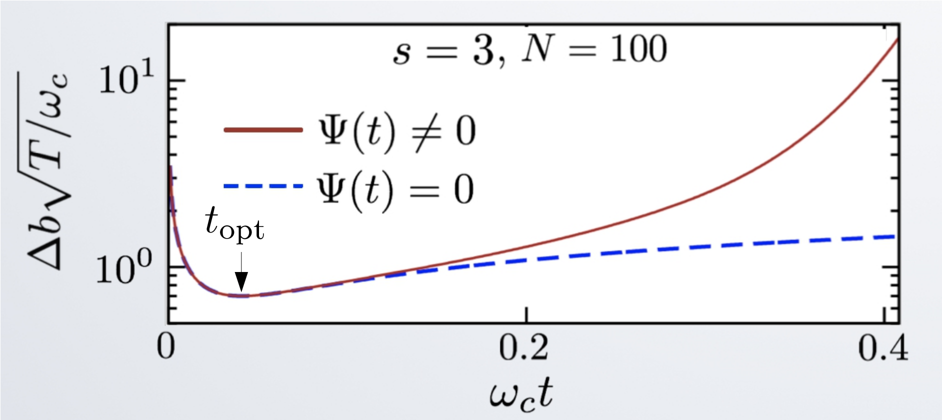
❖ *N* qubit sensors collectively coupled to a bosonic bath in thermal equilibrium:

$$B(t) = 2\sum_k (g_k a_k^\dagger e^{i\Omega_k t} + \text{H.c.}) \qquad |g_k|^2 \to I(\omega) = \alpha\, \omega_c^{1-s} |\omega|^s\, e^{-|\omega|/\omega_c}$$

(1) Initial CSS: A finite Ψ results in significantly increased uncertainty. At short time, optimizing for fixed total evolution time $T = \nu t$ yields asymptotic scaling

$$\Delta\hat{b}_{\text{opt}} = (2\omega_c/T)^{1/2}[\alpha\Gamma(s+1)]^{1/4} N^{-1/4}$$

Worse than SQL



Results: Spin-boson model

❖ *N* qubit sensors collectively coupled to a bosonic bath in thermal equilibrium:

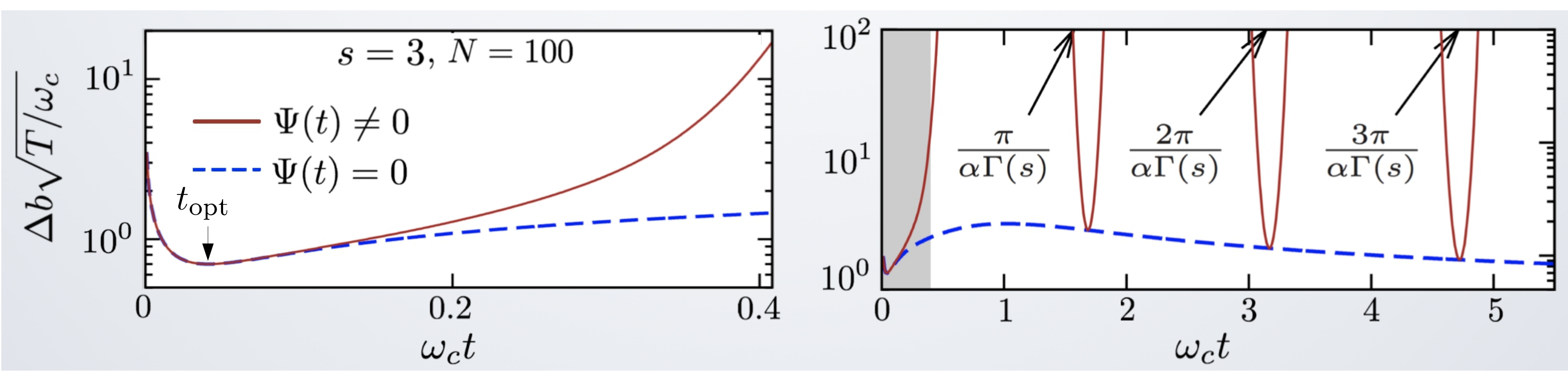
$$B(t) = 2\sum_k (g_k a_k^\dagger e^{i\Omega_k t} + \text{H.c.}) \qquad |g_k|^2 \to I(\omega) = \alpha\, \omega_c^{1-s} |\omega|^s \, e^{-|\omega|/\omega_c}$$

(1) Initial CSS: A finite Ψ results in significantly increased uncertainty. At short time, optimizing for fixed total evolution time $T = \nu t$ yields asymptotic scaling

$$\Delta\hat{b}_{\text{opt}} = (2\omega_c/T)^{1/2} [\alpha\Gamma(s+1)]^{1/4} N^{-1/4}$$

Worse than SQL

- The uncertainty minima at longer times correspond to *disentanglement among the sensors* – the concurrence between any qubit pair vanishes (nearly) exactly at those times.
- The width of the 1st minimum in $\Delta\hat{b}(t)$ wrto t is suppressed as $N^{-1/2}$ ⇒ *Even with perfect knowledge of all noise parameters*, it becomes increasingly hard to minimize $\Delta\hat{b}(t)$ in practice…



Results: Spin-boson model

❖ *N* qubit sensors collectively coupled to a bosonic bath in thermal equilibrium:

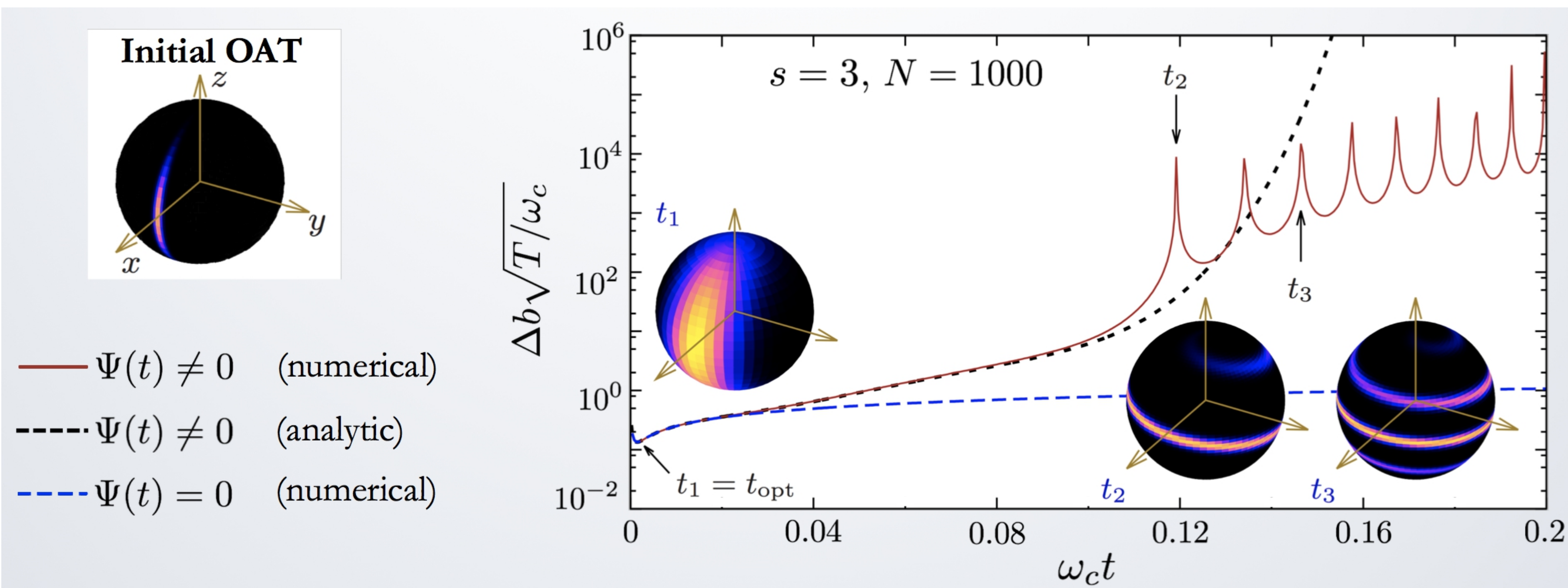
$$B(t) = 2\sum_k (g_k a_k^\dagger e^{i\Omega_k t} + \text{H.c.}) \qquad |g_k|^2 \to I(\omega) = \alpha\, \omega_c^{1-s} |\omega|^s\, e^{-|\omega|/\omega_c}$$

(2) Initial OAT, minimizing initial uncertainty $\Delta J_y(0)$ – At short time, optimizing for fixed total evolution time $T = \nu t$ yields asymptotic scaling

$$\Delta \hat{b}_{\text{opt}} = (4/3)^{1/12} (2\omega_c/T)^{1/2} [\alpha \Gamma(s+1)]^{1/4} N^{-5/12}$$

Better than CSS but not super-classical

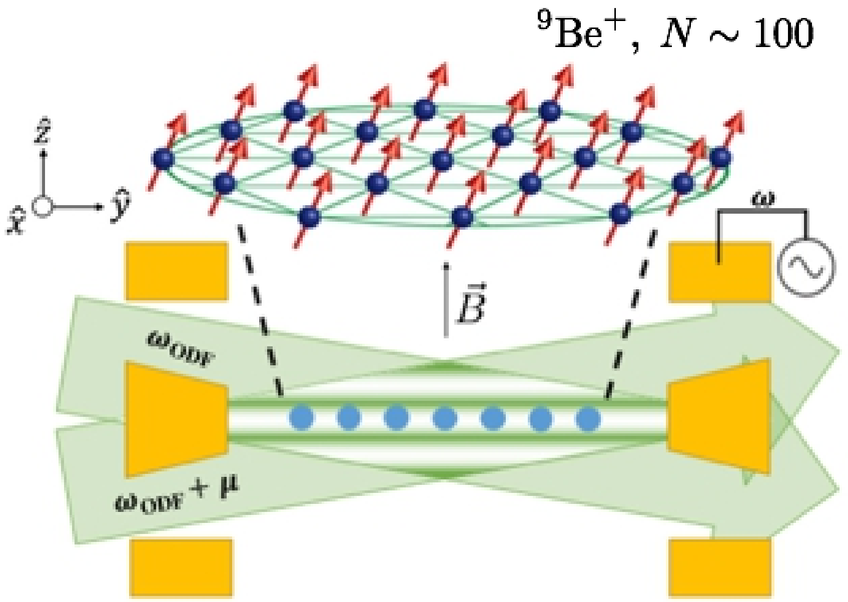
- The optimal [Kitagawa-Ueda] noiseless OATS angles *remain optimal* under dephasing noise.



Results: Trapped ion crystals

❖ Task: Amplitude sensing of classical center-of-mass (CM) lattice oscillations in 2D ion crystal.

Gilmore *et al*, PRL **118** (2017).



A weak RF drive is applied nearly resonant with the CM mode:

$$H_{\rm rf}(t) = \varepsilon \cos(\omega_{\rm rf} t + \delta)\sqrt{\frac{\hbar}{2M\omega_z}}(a_{CM} + a^\dagger_{CM})$$

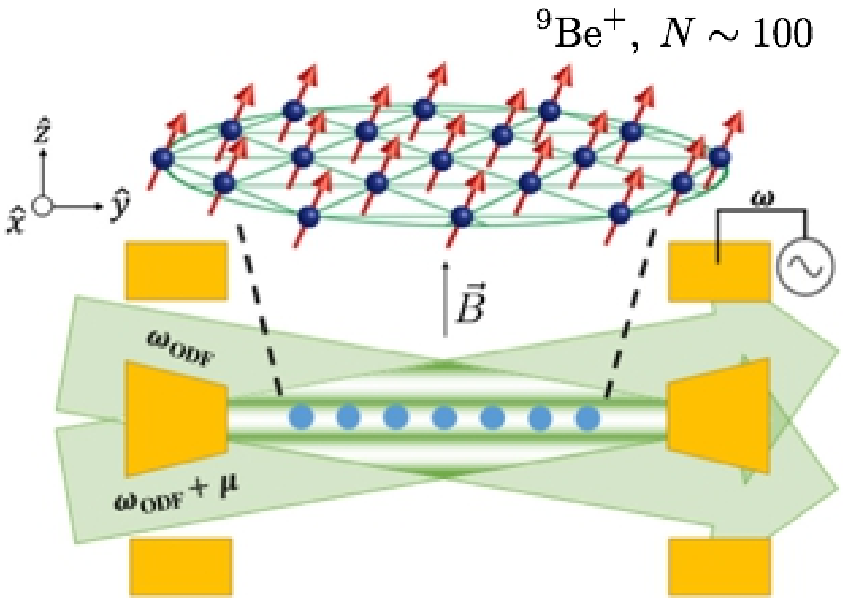
$$\omega_{\rm rf} = \mu \qquad D \equiv \omega_z - \mu \ll \omega_z, \mu$$

Time-dependent optical dipole force couples CM oscillations to collective spin of the ions.

Results: Trapped ion crystals

❖ <u>Task:</u> Amplitude sensing of classical center-of-mass (CM) lattice oscillations in 2D ion crystal.

Gilmore *et al*, PRL **118** (2017).



A weak RF drive is applied nearly resonant with the CM mode:

$$H_{\mathrm{rf}}(t) = \varepsilon\cos(\omega_{\mathrm{rf}}t+\delta)\sqrt{\frac{\hbar}{2M\omega_z}}(a_{CM}+a_{CM}^\dagger)$$

$$\omega_{\mathrm{rf}} = \mu \qquad D \equiv \omega_z - \mu \ll \omega_z, \mu$$

Time-dependent optical dipole force couples CM oscillations to collective spin of the ions.

❖ Assuming that CM is far-detuned from all other modes, and invoking RWA, the problem maps to frequency estimation *under appropriate time-dependent modulation functions*:

$$H_{SB}(t) \approx \left[\frac{U\delta k Z_c}{2}(1-\cos Dt) + \cos\mu t\, B_{\mathrm{CM}}(t)\right]\sum_n \sigma_n^z$$

$$B_{\mathrm{CM}}(t) = \frac{2U\,\delta k}{\hbar}\sqrt{\frac{\hbar}{2MN\omega_z}}(a_{\mathrm{CM}}e^{-i\omega_z t} + \mathrm{H.c.}) \qquad \Psi(t) \approx \left(\frac{U\delta k}{\hbar}\sqrt{\frac{\hbar}{MN\omega_z}}\right)^2 \frac{\omega_z t\,(1-\mathrm{sinc}Dt)}{\mu^2-\omega_z^2}$$

Results: Trapped ion crystals

❖ Task: Amplitude sensing of classical center-of-mass (CM) lattice oscillations in 2D ion crystal.

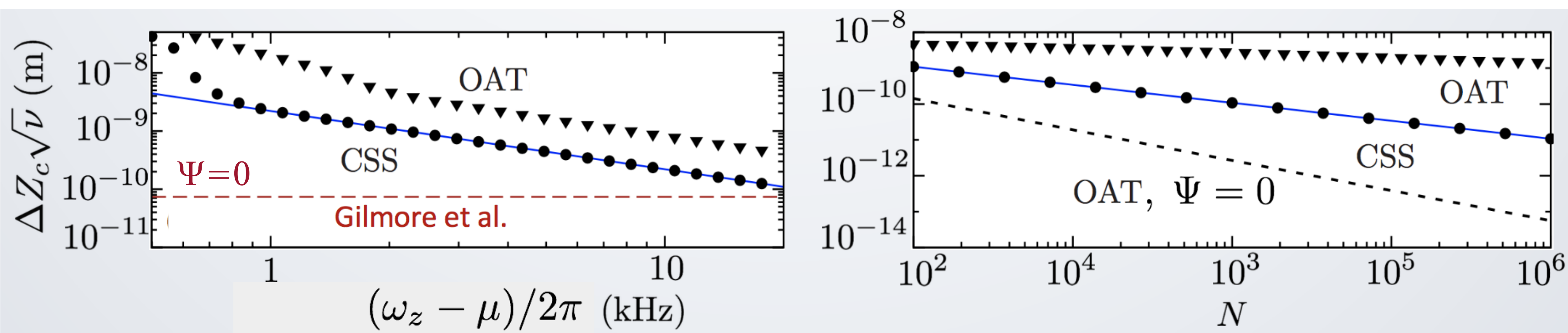
$$H_{SB}(t) \approx \left[\frac{U\delta k Z_c}{2}(1-\cos Dt) + \cos\mu t\, B_{\rm CM}(t)\right]\sum_n \sigma_n^z$$

➡ The uncertainty is now optimized at a fixed number of experimental shots, $\nu = T/t$.

(1) Initial CSS: Due to finite $\Psi(t)$, driving near resonance with CM yields uncertainty that exceeds significantly the one estimated by neglecting quantum noise.

(2) Initial OAT, minimizing initial uncertainty $\Delta J_y(0)$: The uncertainty is even larger than SQL, due to the fact that *the state becomes anti-squeezed by the noise* ⇒ Quantum correlations have a detrimental effect on precision!

$\Delta Z_c \propto N^{-1/6}$ vs. $\Delta Z_c \propto N^{-5/6}$ $(\Psi = 0)$



[Ongoing] Countering non-Markovian collective noise?

- The quantum nature of the noise brings about additional unexpected features...
  - ➡ Thanks to symmetry, GHZ state is *insensitive to quantum noise*: $\Psi(t)$ does not enter the reduced dynamics ⇒ effectively classical, correlated noise,

$$\Delta\hat{b}_{\min}(t) = 1/\sqrt{F_Q} \quad \Rightarrow \quad F_Q = N^2 e^{-2N^2 \chi(t)}$$

    (Q1) Can the use of dynamical control/filter-shaping techniques *suppress decay*, while retaining a favorable signal-to-noise ratio for metrological advantage?...

  - ➡ Quantum noise enters the reduced dynamics *unitarily:* $\rho_S(t) = e^{-i\Psi(t)J_z^2}[\,\rho_S(t)|_{\Psi=0}\,]e^{i\Psi(t)J_z^2}$.
    Since the QFI is unitarily invariant, $\Delta\hat{b}_{\min} = \Delta\hat{b}_{\min}|_{\Psi=0}$

    (Q2) Can measurement(s) that optimally *cancels the effects of quantum noise* be constructed?...

[Ongoing] Countering non-Markovian collective noise?

- The quantum nature of the noise brings about additional unexpected features...
  - ➞ Thanks to symmetry, GHZ state is *insensitive to quantum noise*: $\Psi(t)$ does not enter the reduced dynamics ⇒ effectively classical, correlated noise,

$$\Delta\hat{b}_{\rm min}(t) = 1/\sqrt{F_Q} \quad \Rightarrow \quad F_Q = N^2 e^{-2N^2 \chi(t)}$$

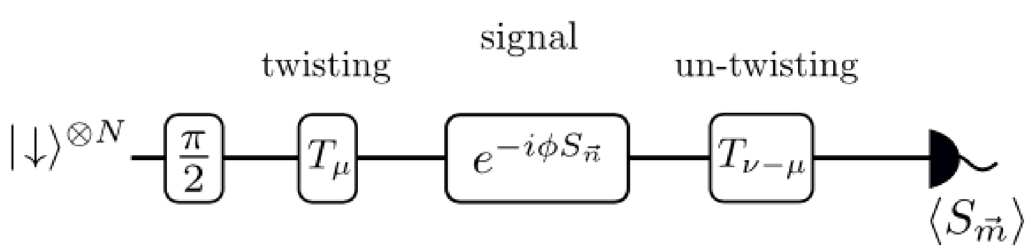
(Q1) Can the use of dynamical control/filter-shaping techniques *suppress decay*, while retaining a favorable signal-to-noise ratio for metrological advantage?...

  - ➞ Quantum noise enters the reduced dynamics *unitarily:* $\rho_S(t) = e^{-i\Psi(t)J_z^2}[\,\rho_S(t)|_{\Psi=0}\,]e^{i\Psi(t)J_z^2}$.
    Since the QFI is unitarily invariant, $\Delta\hat{b}_{\rm min} = \Delta\hat{b}_{\rm min}|_{\Psi=0}$

(Q2) Can measurement(s) that optimally *cancels the effects of quantum noise* be constructed?...

- *Detailed knowledge about the noise* may be exploited in principle to adaptively modify the measurement protocol, while also altering the probe dynamics with external control

(Q3) Expand and further generalize *twisting-echoes protocols* to noisy settings?...



Schulte *et al*, Quantum **4** (2020).

[Ongoing] Non-collectivity as a resource?

- N qubit sensors coupled to a bosonic bath in thermal equilibrium:

$$B(t) = 2\sum_k (g_k a_k^\dagger e^{i\Omega_k t} + \mathrm{H.c.}) \mapsto B_n(t) = 2\sum_k (\underline{g_k e^{i\vec{k}\cdot\vec{r}_n}} a_k^\dagger e^{i\Omega_k t} + \mathrm{H.c.}), \quad \Omega_k = v|\vec{k}| = vk$$

Position-dependent coupling

➞ Case study: Randomize sensor's spatial location – make sensors i.i.d. according to

$$P(\vec{r}_n) = \frac{e^{-r_n^2/2\epsilon^2}}{(2\pi\epsilon^2)^{d/2}}, \quad d \in \{1,2,3\}, \qquad \mathbb{P}(\vec{r}_1, \ldots, \vec{r}_N) = \prod_n P(\vec{r}_n)$$

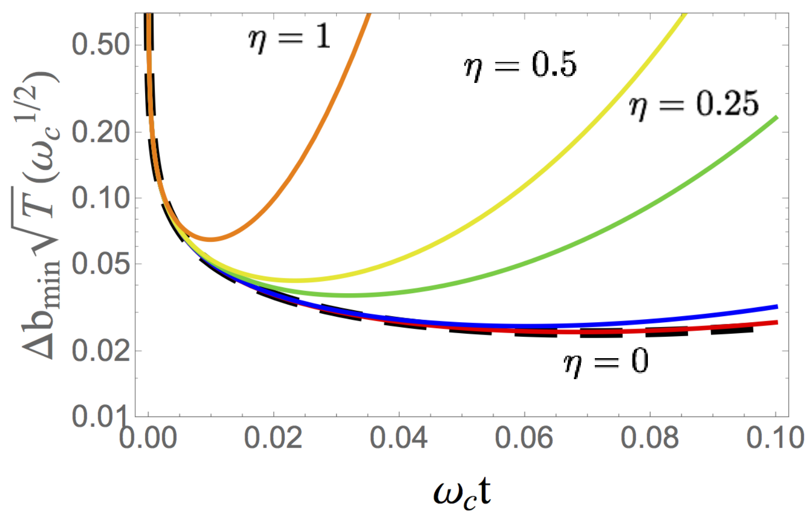
Averaging over sensors' locations effectively suppresses noise spatial correlations,

$$\mathbb{E}\{\mathrm{Tr_B}\{B_n(t)B_m(0)\rho_B\}\}_{\vec{r}} \to \delta_{nm} f_n(t) \quad \eta \equiv v/(\epsilon\omega_c) \ll 1$$

- *Super-classical precision scaling may be recovered* – in principle – for initial entangled states:

OAT scaling: $\Delta\hat{b}_{\min} \propto N^{-2/3}$

GHZ scaling: $\Delta\hat{b}_{\min} = \Delta\hat{b}_{\mathrm{opt}} \propto N^{-3/4}$ **Zeno scaling**



Riberi, Norris, Beaudoin & LV, forthcoming (2021).

Conclusion

❖ Characterizing and controlling realistic quantum systems calls for modeling open dynamics beyond simplifying assumptions – by acknowledging *nontrivial spatiotemporal correlations and non-classicality* of the noise environment. Additional complexity may stem from

- ➡ Initial *non-factorization* – need improved dynamical representations…

  Paz-Silva *et al*, PRA **100** (2019); Alipour *et al*, PRX **10** (2020).

- ➡ *Non-Gaussianity* – need higher-order correlators and polyspectra…
- ➡ *Non-stationarity* – need filter functions/quantum noise spectroscopy beyond frequency domain…

  Norris *et al*, PRL **116** (2016); Chalermpusitarak *et al*, arXiv:2008.13216.

❖ Spatiotemporally correlated quantum noise introduces *additional uncertainty* in quantum parameter estimation, due to bath-mediated entanglement and squeezing among sensors. *Quantitative noise knowledge* becomes a prerequisite for tailoring noise-mitigation protocols…

❖ Additional work is needed to assess *metrological impact* of spatiotemporally correlated noise:

- ➡ Other physical settings: Amplitude sensing *beyond collective regime?*… Atomic clocks?…
- ➡ Once noise is characterized, *what dictates ultimate precision bound?*…
- ➡ Can schemes for QEC-sensing be extended to non-Markovian regime?…
- ➡ To what extent *can external control or QEC help restore metrological advantage?*…