IICQI 2007

Spin network communication protocols

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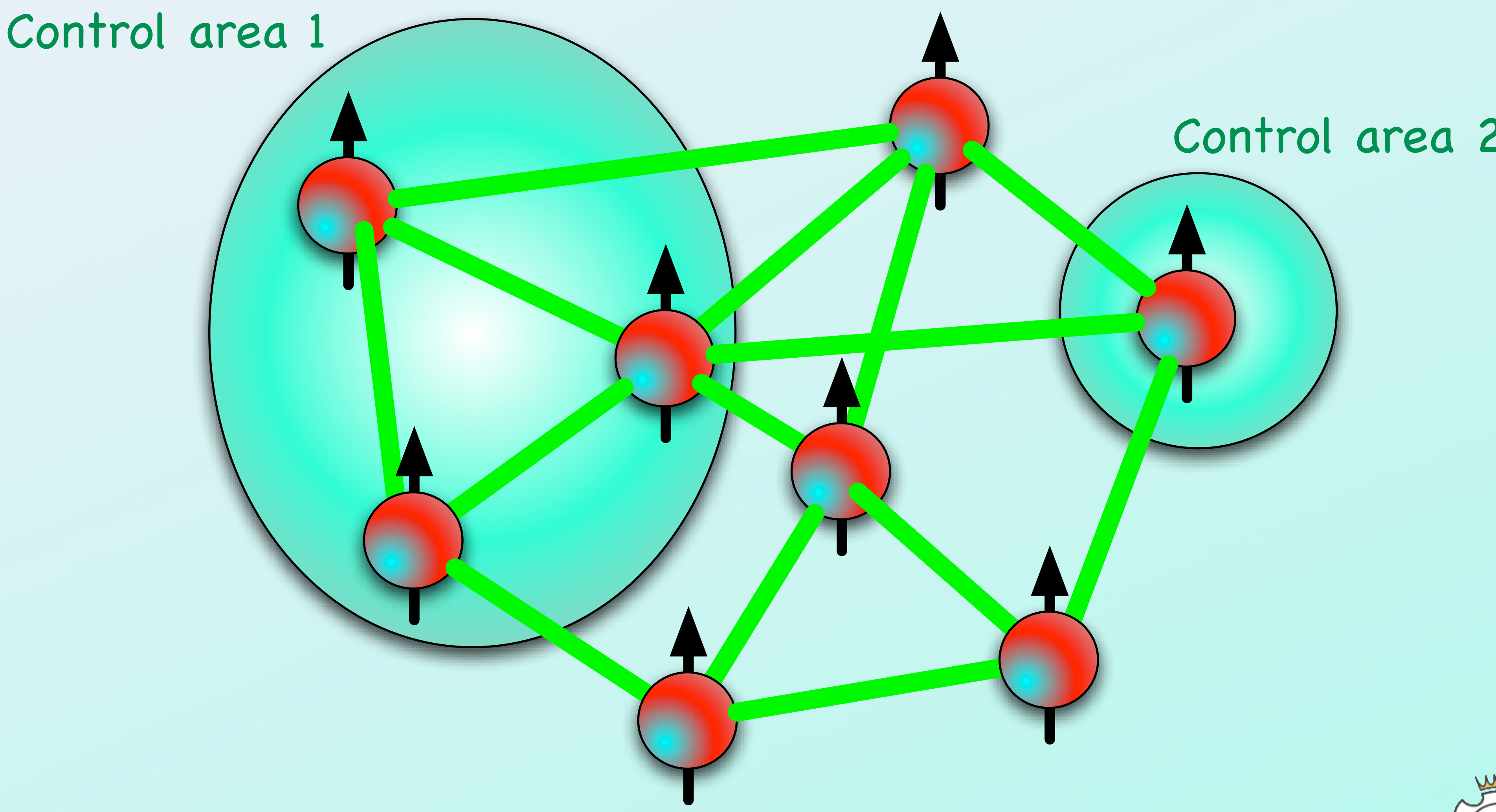
NEST-CNR-INFM & Scuola Normale Superiore (PISA)

NEST National Enterprise for nanoScience and nanoTechnology

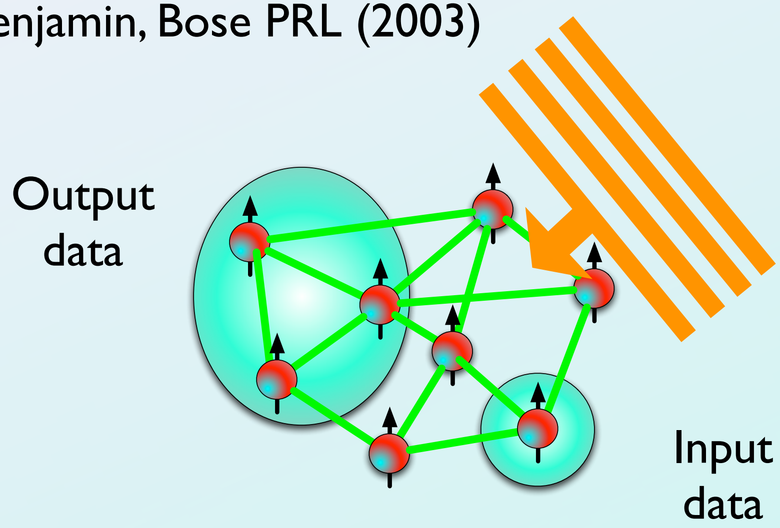
Spin Network = Collection of 2-dim systems coupled through some given Hamiltonian



Examples of quantum protocols

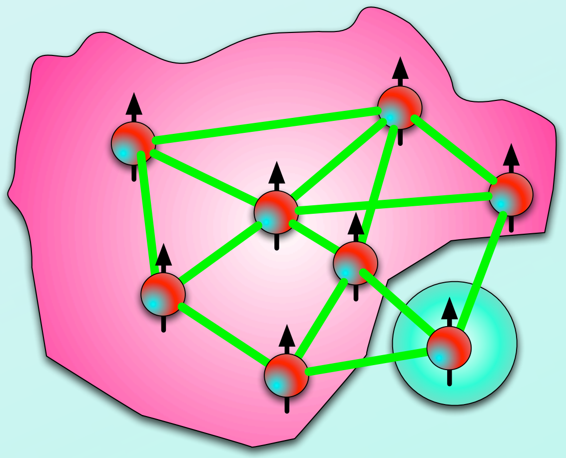
Universal Quantum computation

DiVincenzo et al. Nature(2000)
Benjamin, PRL88 (2002)
Benjamin, Bose PRL (2003)
...

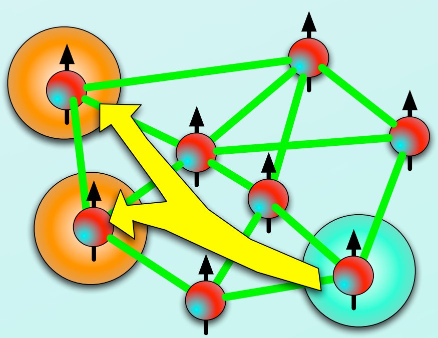


Quantum control:

quantum state preparation, cooling, read-out



Fitzsimons et al. PRL99 (2007)
Burgarth, VG PRL (to be published) (2007)



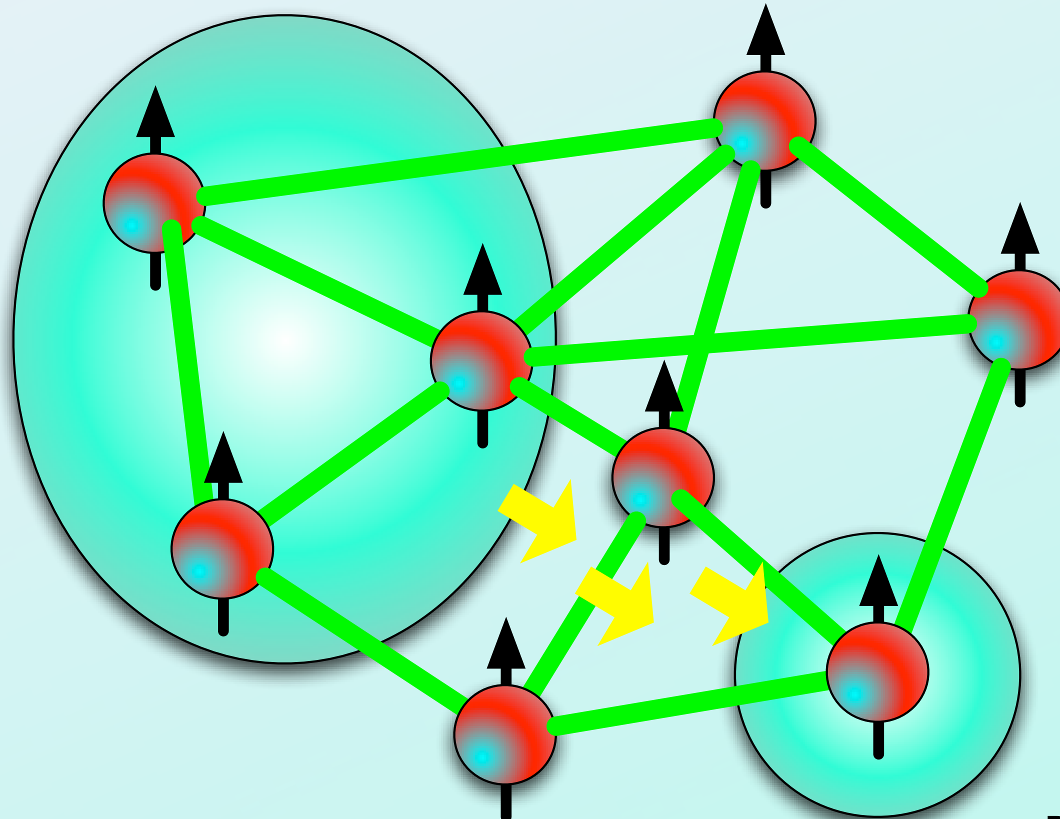
(approx) Quantum cloning

De Chiara, et al. PRA (2004)

Quantum communication (transmission of a state)

“spin chain quantum communication scheme”

Alice
(the sender)



Bob
(the receiver)

MOTIVATIONS

Quantum Information protocol with "minimal" external control

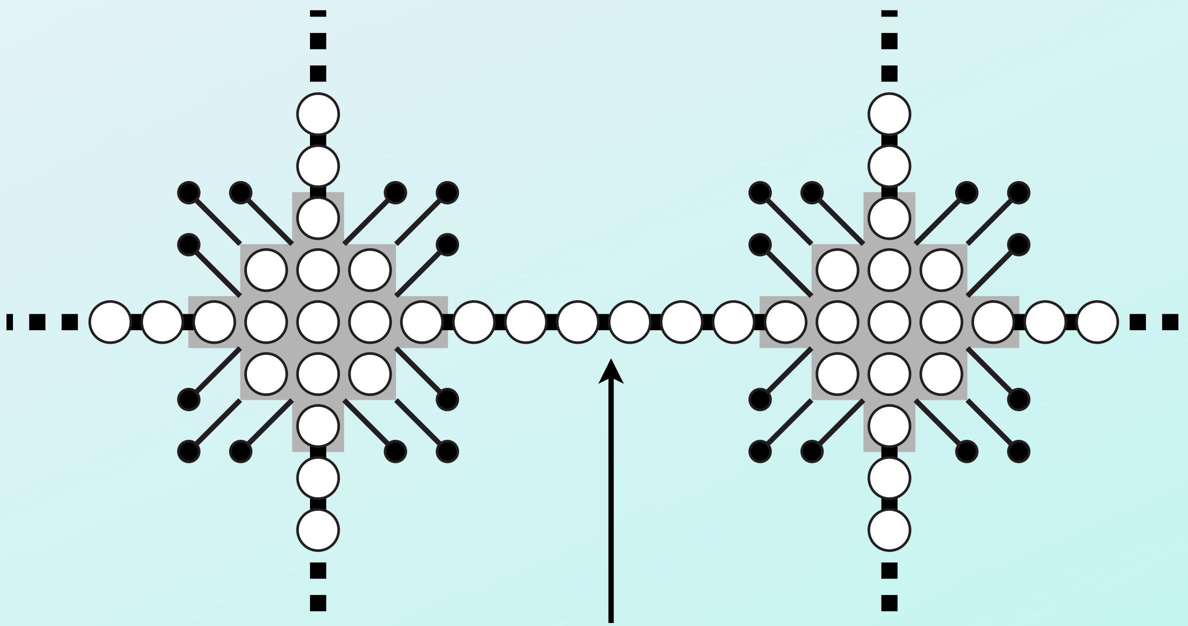
Choose a given model and use just the time evolution
(less flexible but more stable)

Implementation of Quantum Information schemes in solid state devices

- Josephson arrays
- Paul Traps
- Optical lattices
- QED+atoms
- Penning Traps
- Quantum dots
- NMR

New methods to analyze many-body physics

Not suited for long distance communication but potentially useful to connect *clusters of quantum registers* (for reasons of compactness, mobility and *cost this may be preferable than a single HUGE register*). *Also QEC is not linear in the dimension of the register (the amount of control to protect a register of N+M qubits, is arguably bigger than the control required to control two registers of N and M qubits).*



quantum register I

Switchable couplings (lot of classical control)

quantum data-bus

No switchable couplings (less control, less susceptible to errors)

quantum register II

Switchable couplings (lot of classical control)

“Optimal quantum-chain communication by end gates”

Phys. Rev. A **75**, 062327 (2007)

D. Burgarth, ETH (Zurich)

V. G.

S. Bose, University College (London)

“Full control by locally induced relaxation”

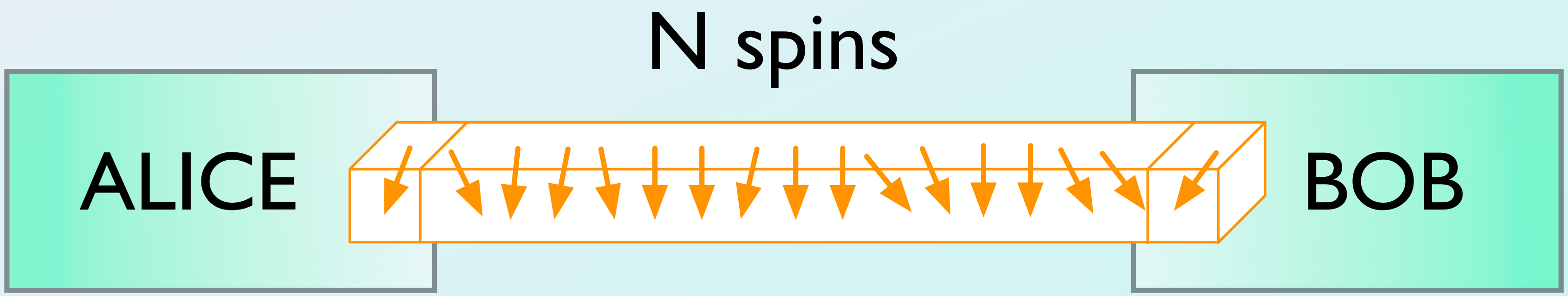
Phys. Rev. Lett. (to appear)

D. Burgarth, ETH (Zurich)

V. G.

Quantum Chain model for communication

Bose S 2003 *Phys. Rev. Lett.* **91** 207901



Linear chain of permanently coupled spins

Ferromagnetic Heisenberg interaction

Ferromagnetic Heisenberg coupling *

$$\boxed{H = -\sum_{\langle i,j\rangle} J_{ij}(\sigma_x^i\sigma_x^j + \sigma_y^i\sigma_y^j + \sigma_z^i\sigma_z^j) - \sum_{i=1}^{N} B_i\sigma_z^i}$$

$$[H, S_z^{(tot)}] = 0$$

z-axis component of the total spin preserved

Spin sectors (i.e. total number N_s of spins up) are preserved by the Hamiltonian evolution

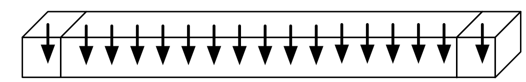
$N_s = 0$	$\vert\vec{0}\rangle \equiv \vert\downarrow\downarrow\cdots\downarrow\rangle \longrightarrow \vert\vec{0}\rangle$
$N_s = 1$	$\vert\vec{j}\,\rangle \equiv \vert\downarrow\downarrow\downarrow\cdots\downarrow\uparrow\downarrow\cdots\downarrow\rangle \rightarrow \sum_{j'} f_{j',j}(t)\vert\vec{j'}\,\rangle$ $f_{j',j}(t) = \langle\vec{j'}\vert e^{-iHt/\hbar}\vert\vec{j}\,\rangle$ transmission amplitudes

* similar results applies also for XXZ, XX couplings

Transferring the qubit state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

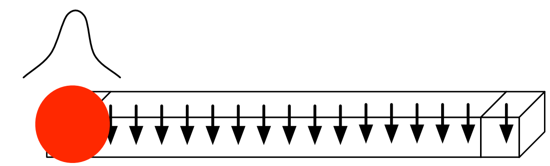
Bose S 2003 *Phys. Rev. Lett.* **91** 207901

0) Chain is initialized in the stationary state $|\vec{0}\rangle$



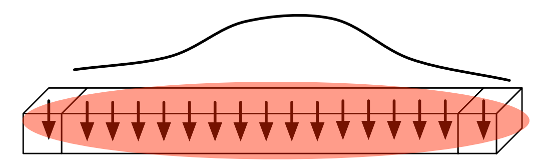
i) At time t=0 Alice prepares (instantaneously) her spin in the input state

$$|\Psi(0)\rangle = (\alpha|\downarrow\rangle + \beta|\uparrow\rangle) \otimes |\downarrow\cdots\downarrow\rangle = \alpha|\vec{0}\rangle + \beta|\vec{1}\rangle$$

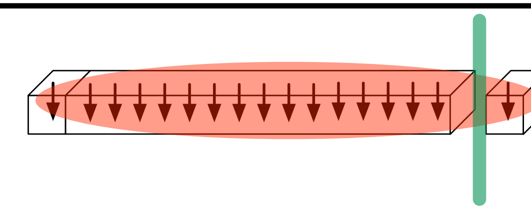


ii) The chain freely evolves for a time t

$$|\Psi(t)\rangle = \alpha|\vec{0}\rangle + \beta\sum_{j=1}^{N} f_{j,1}(t)|\vec{j}\rangle$$



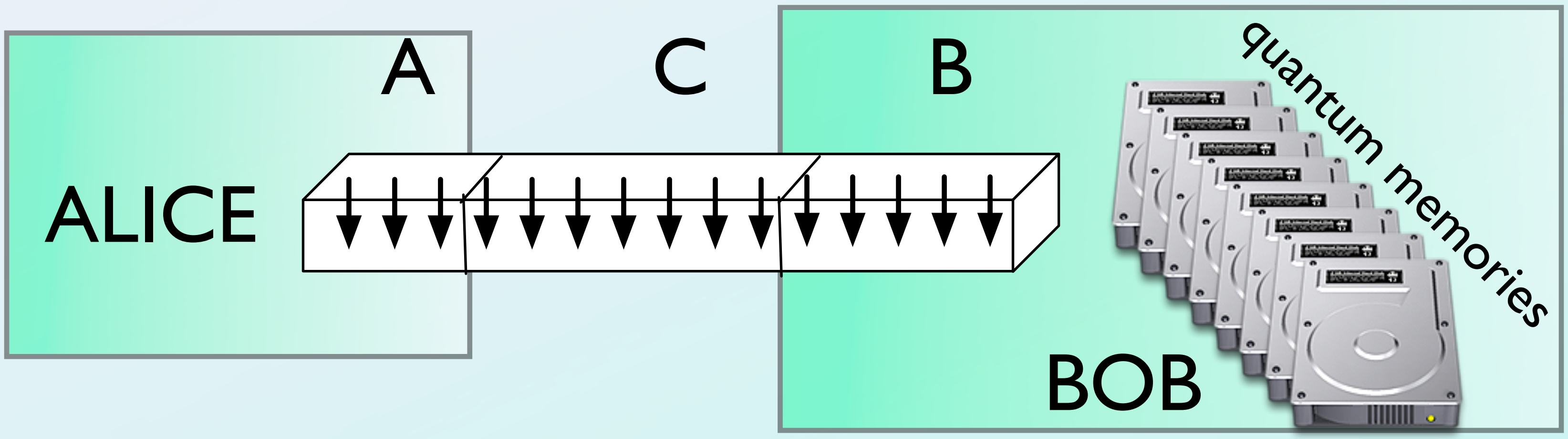
iii) Bob (instantaneously) disconnects the last spin from the chain

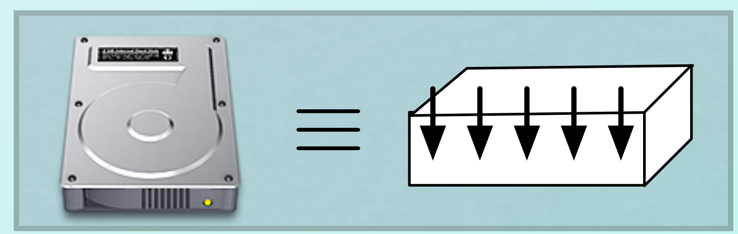


Perfect Transfer if $|\Psi(t)\rangle = \alpha|\vec{0}\rangle + \beta|\vec{N}\rangle \;\; = |\downarrow\cdots\downarrow\rangle \otimes (\alpha|\downarrow\rangle + \beta|\uparrow\rangle)$

Memory protocol

VG and D. Burgarth PRL **96,** 030501 (2006)





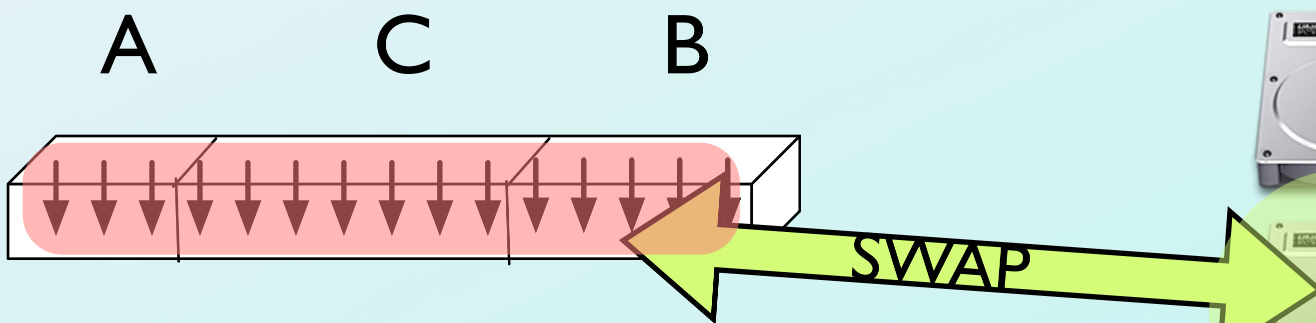
ENCODING

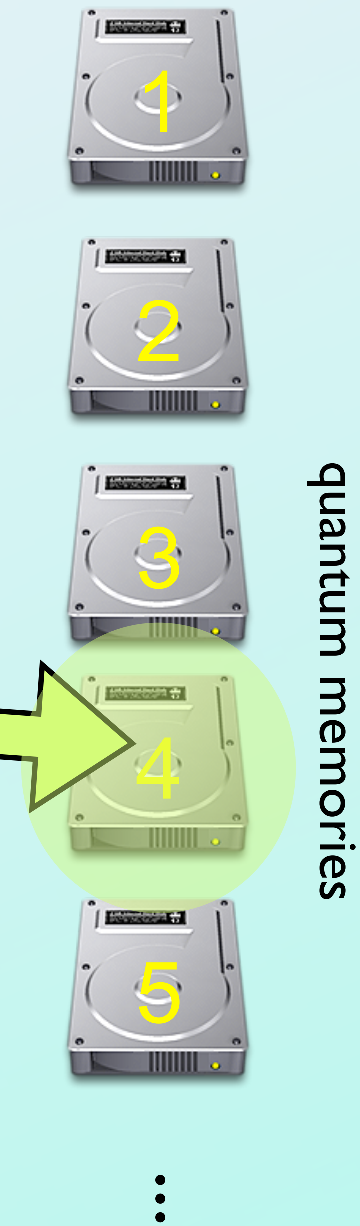
Alice controls Na spins and she is allowed to encode in them up to Na qubits of information

DECODING

At regular time intervals Bob applies SWAPS ops to transfer the state of the B spins in to his quantum memories

$$t \approx NT_e(\ln N_A + |\ln(1-F)|)/N_B$$



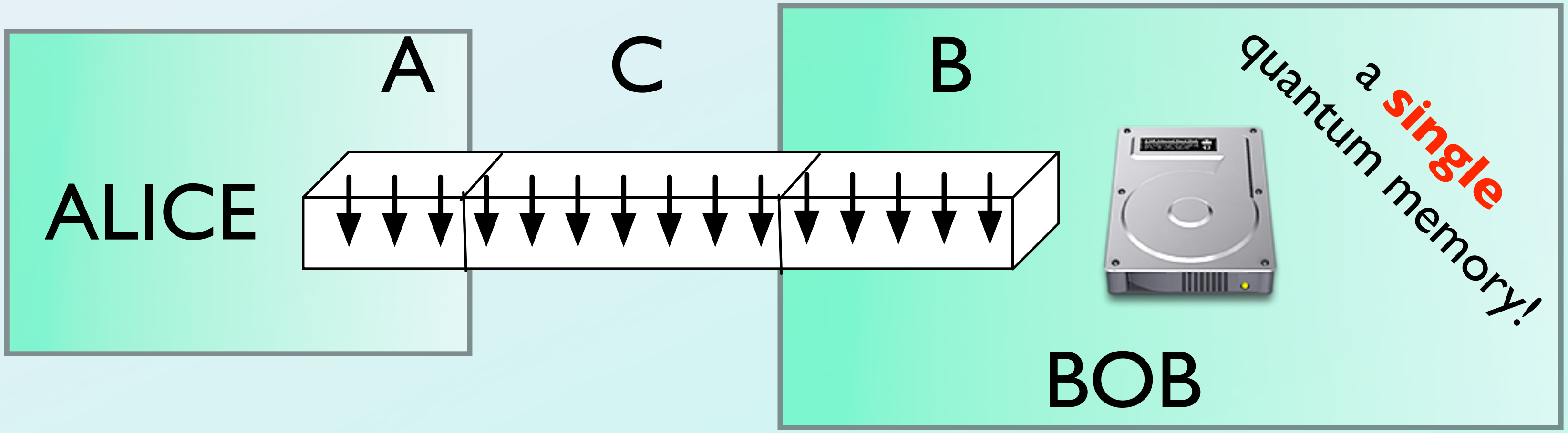


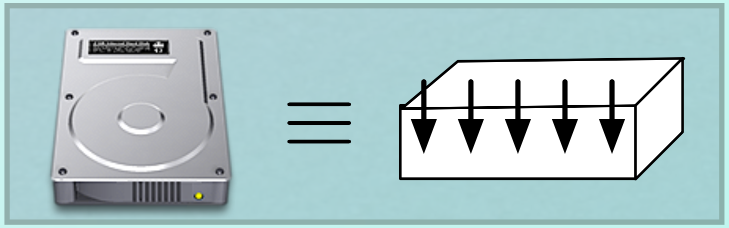
Since the whole process is unitary this is a “coherent cooling” of the chain (similar to homogezenization*). We are indeed driving the ACB towards its ground state. The information originally contained in AC is transferred to the memory array!

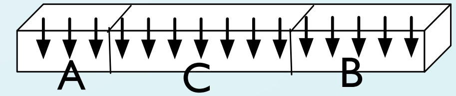
* M. Ziman *et al.*, Phys. Rev. A **65**, 042105 (2002).

End gates protocol

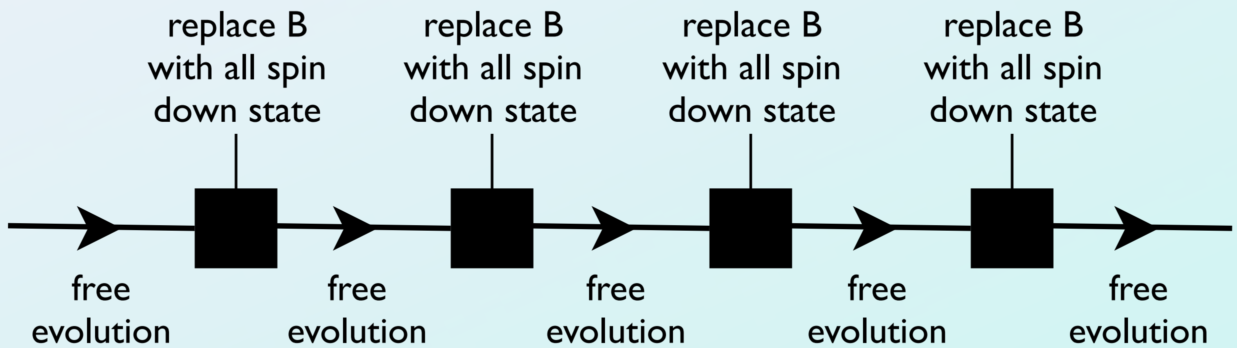
Burgarth, VG, Bose   PRA75 (2007)



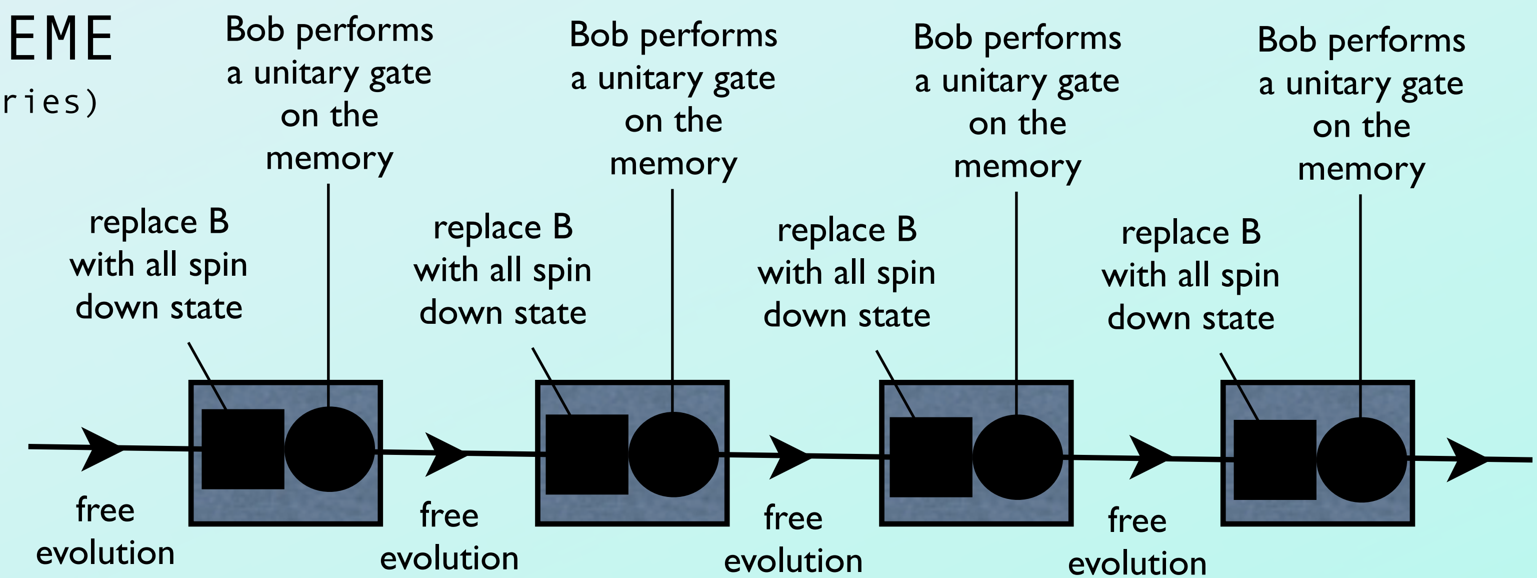


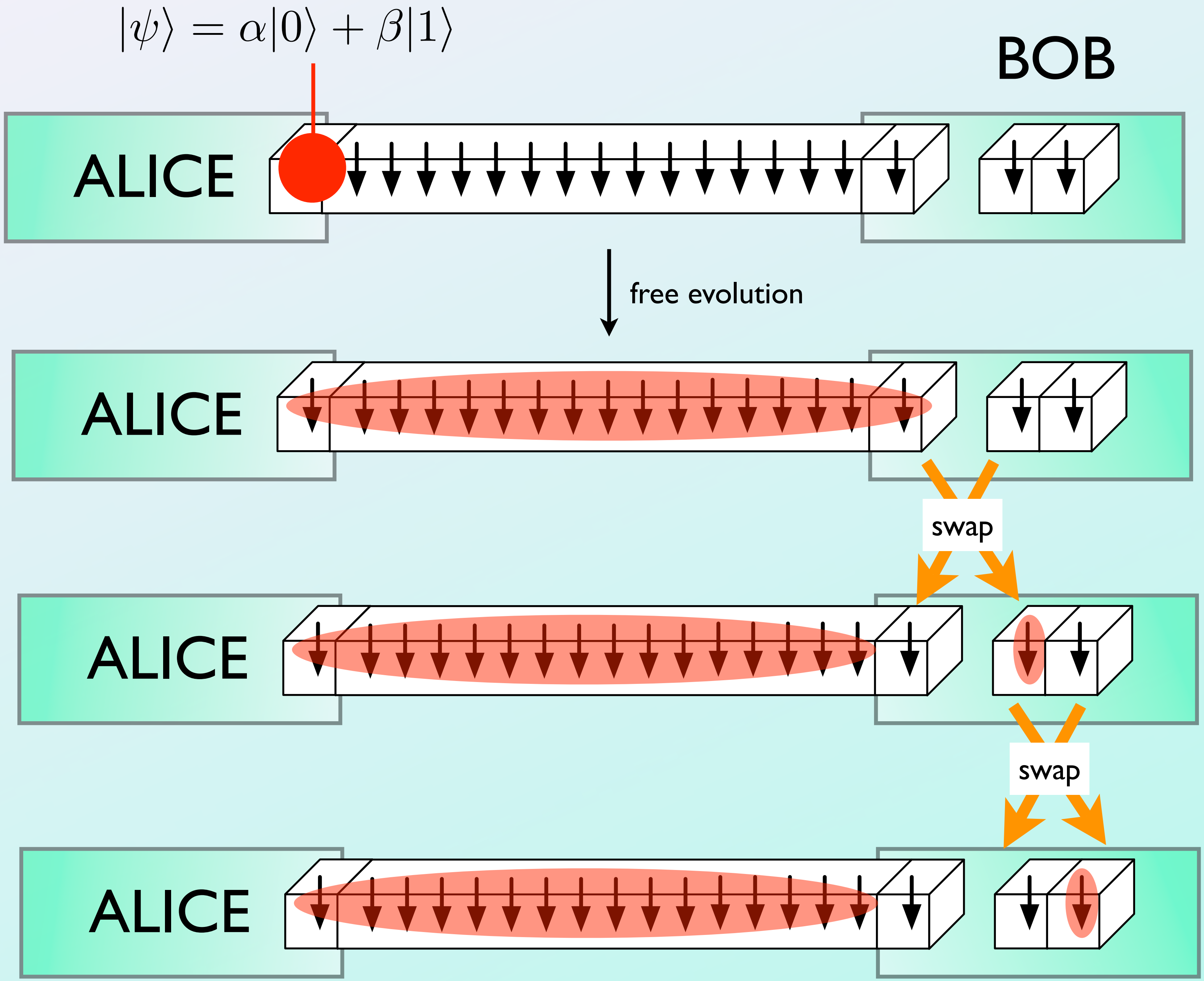


OLD SCHEME (infinite memories)



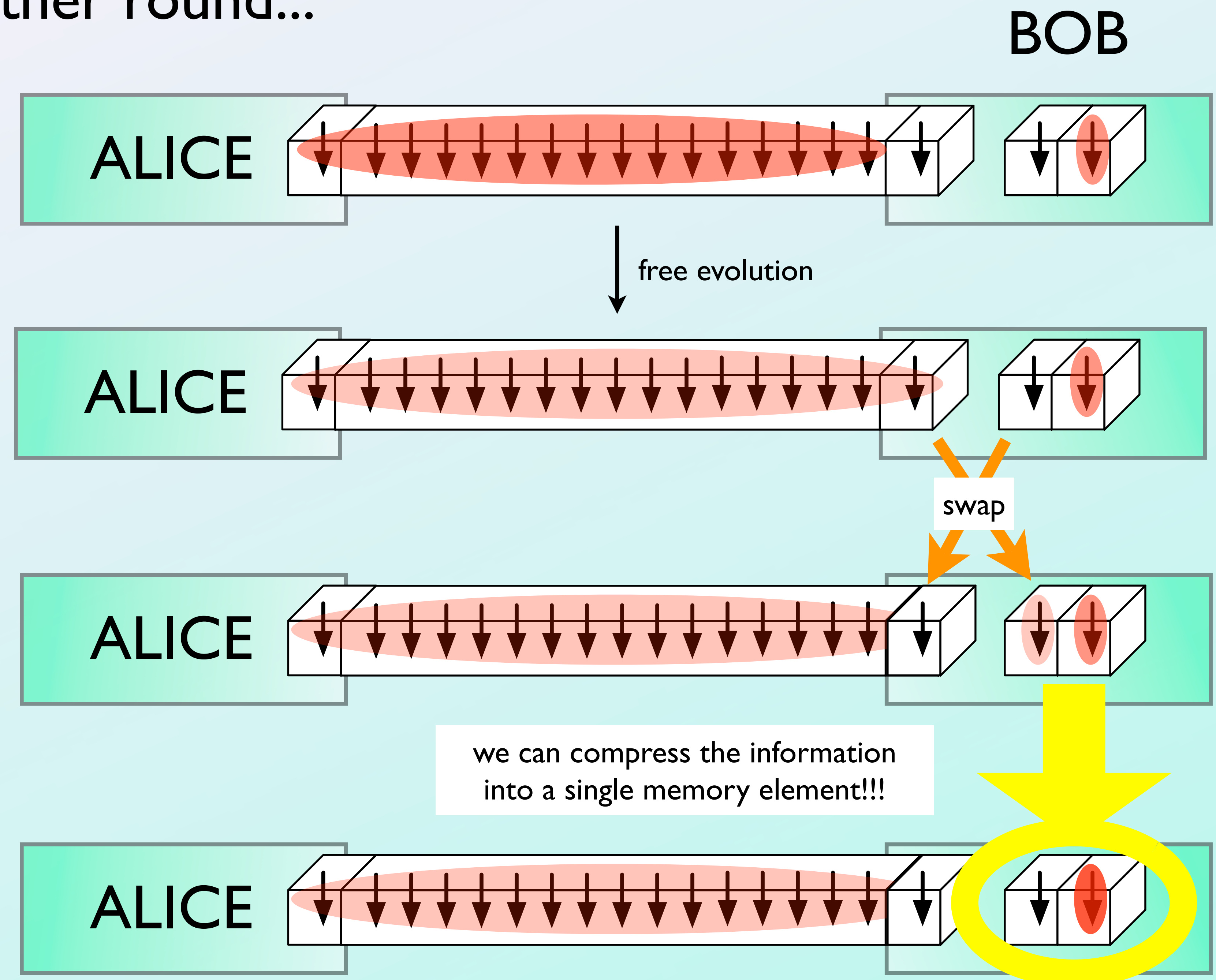
NEW SCHEME (finite memories)





$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$
BOB
ALICE
free evolution
ALICE
swap
ALICE
swap
ALICE

another round...



go for the third one...

$$\gamma_{ij} = f_{i,j}(t)$$

Free evolution of the chain

$$\Big(\alpha|000...000\rangle + \beta \sum_{n_0=1,N} \gamma_{1,n_0}|n\rangle\Big) \otimes |00\rangle_M$$

swap between last element and first memory

$$\alpha|000...000\rangle \otimes |00\rangle_M + \beta \sum_{n_0=1,N-1} \gamma_{1,n_0}|n_0\rangle \otimes |00\rangle_M + \beta\gamma_{1N}|000...000\rangle \otimes |10\rangle_M$$

$$= |000...000\rangle \otimes (\alpha|00\rangle_M + \beta\gamma_{1N}|10\rangle_M) + \beta \sum_{n_0=1,N-1} \gamma_{1,n}|n_0\rangle \otimes |00\rangle_M$$

swap between first memory and second memory

$$|000...000\rangle \otimes (\alpha|00\rangle_M + \beta\gamma_{1N}|01\rangle_M) + \beta \sum_{n_0=1}^{N-1} \gamma_{1,n}|n_0\rangle \otimes |00\rangle_M$$

Free evolution of the chain

$$|000...000\rangle \otimes (\alpha|00\rangle_M + \beta\gamma_{1N}|01\rangle_M) + \beta \sum_{n_0=1}^{N-1} \sum_{n_1=1}^{N} \gamma_{1,n_0}\gamma_{n_0,n_1}|n_1\rangle \otimes |00\rangle_M$$

$$|000...000\rangle \otimes \left[\ \alpha|00\rangle_M + \beta\gamma_{1N}|01\rangle_M + \beta\sum_{n_0=1}^{N-1}\gamma_{1,n_0}\gamma_{n_0,N}|10\rangle_M\right] + \beta\sum_{n_0=1}^{N-1}\sum_{n_1=1}^{N-1}\gamma_{1,n_0}\gamma_{n_0,n_1}|n_1\rangle\otimes|00\rangle_M$$

$$= |000...000\rangle \otimes \left[\ \alpha|00\rangle_M + \beta\sqrt{\eta_1}|\phi_1\rangle_M\right] + \beta\sum_{n_0=1}^{N-1}\sum_{n_1=1}^{N-1}\gamma_{1,n_0}\gamma_{n_0,n_1}|n_1\rangle\otimes|00\rangle_M\ ,$$

$$\eta_1 = |\gamma_{1,N}|^2 + |\textstyle\sum_{n_0=1}^{N-1}\gamma_{1,n_0}\gamma_{n_0,N}|^2$$

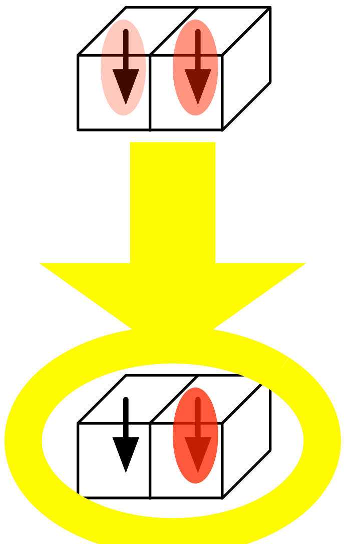
$$|\phi_1\rangle_M = \left[\gamma_{1N}|01\rangle_M + \sum_{n_0=1}^{N-1}\gamma_{1,n_0}\gamma_{n_0,N}|10\rangle_M\right]/\sqrt{\eta_1}.$$

this is orthogonal with respect to $|00\rangle_M$!!!!!

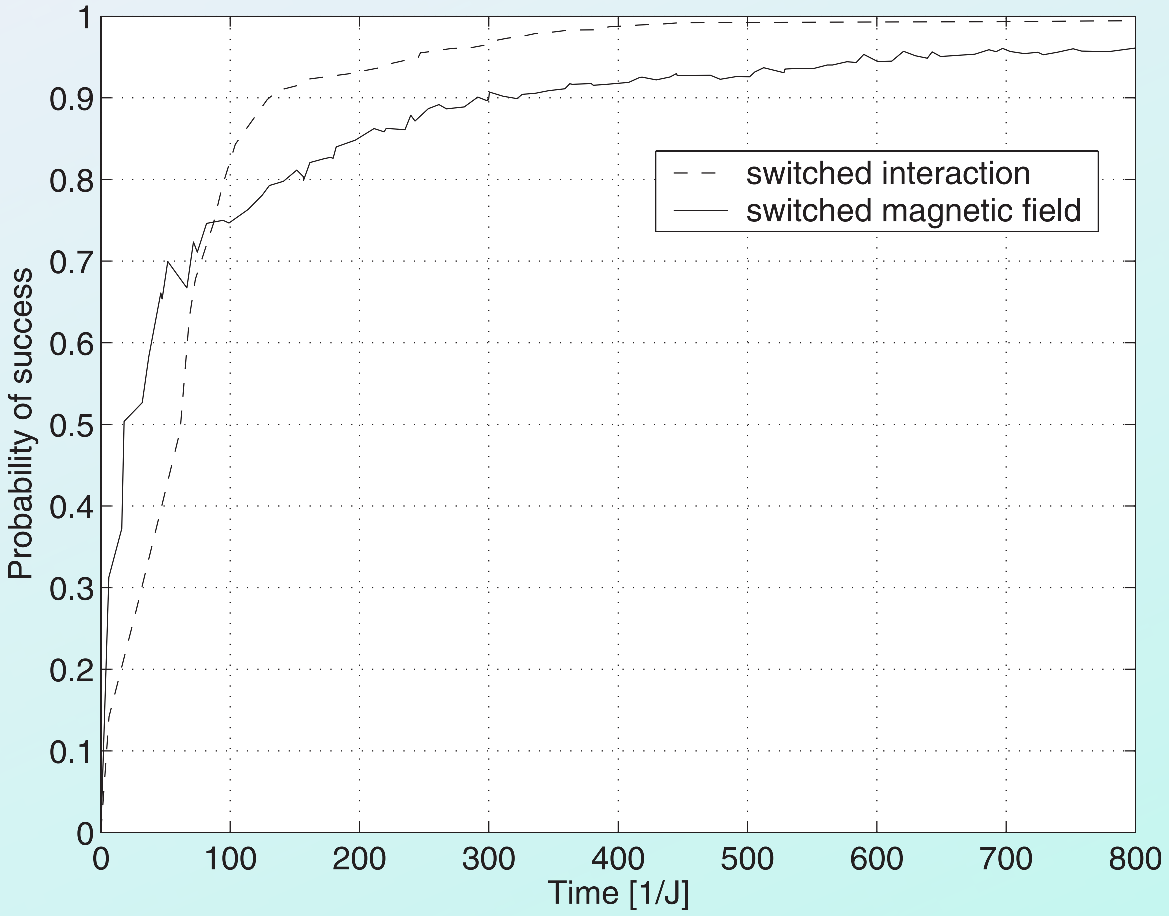
Therefore we can define a two-qubit unitary operator which performs the following transformation:

$$V_1|\phi_1\rangle_M = |01\rangle_M$$
$$V_1|00\rangle_M = |00\rangle_M\ .$$

This is our compression gate

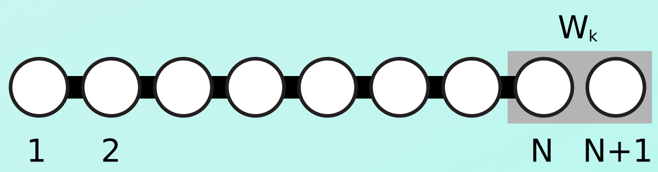


Simplification



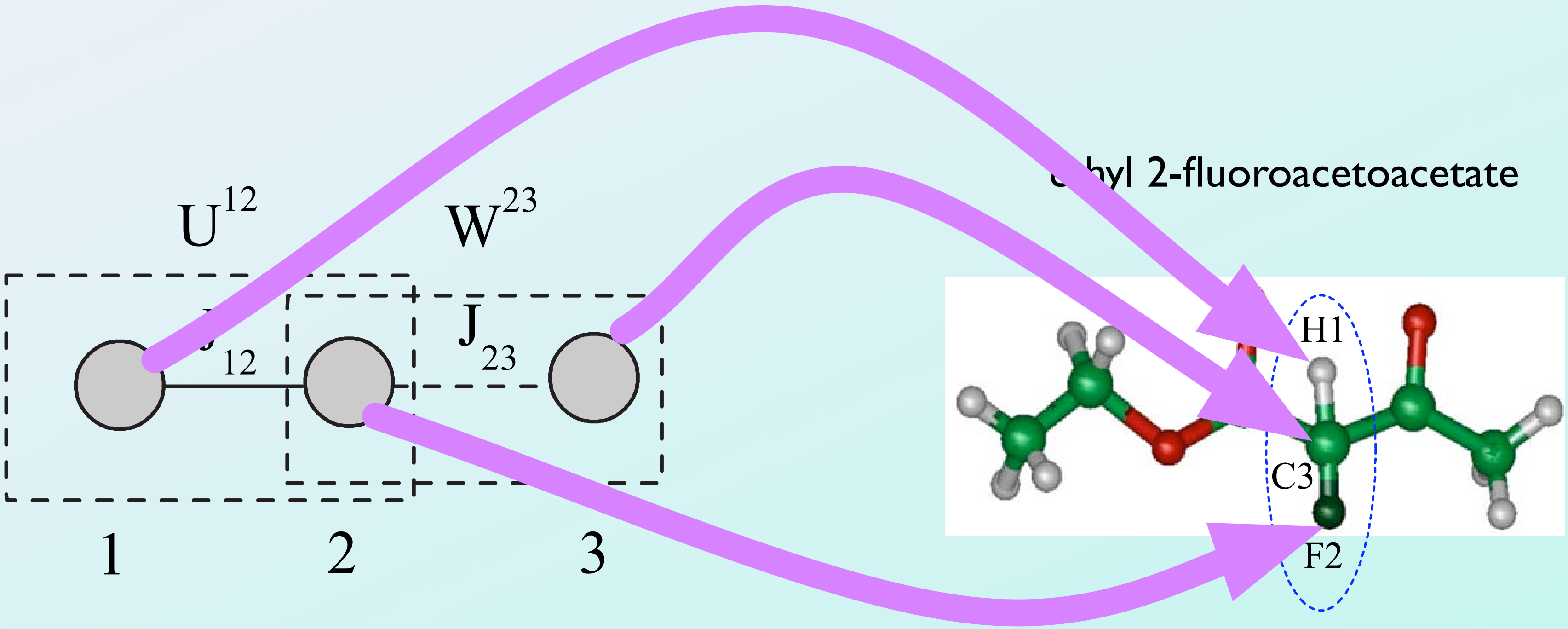
$$H(t) = J\sum_{n=1}^{N} \sigma_n^- \sigma_{n+1}^+ + \mathrm{H.c.} + B\Delta(t)\sigma_{N+1}^z$$

$$H(t) = J\sum_{n=1}^{N-1} \sigma_n^- \sigma_{n+1}^+ + \Delta(t)\sigma_N^- \sigma_{N+1}^+ + \mathrm{H.c.}$$



NMR implementation of the "end gates" protocol (N=2)

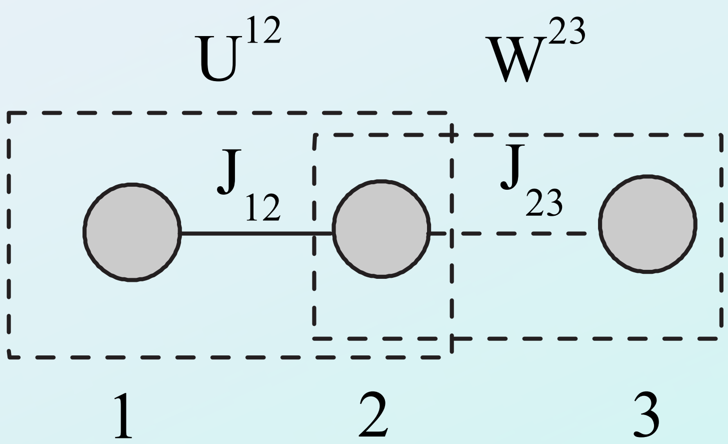
Zhang, Rajendran, Peng and Suter PRA 76 (2007)

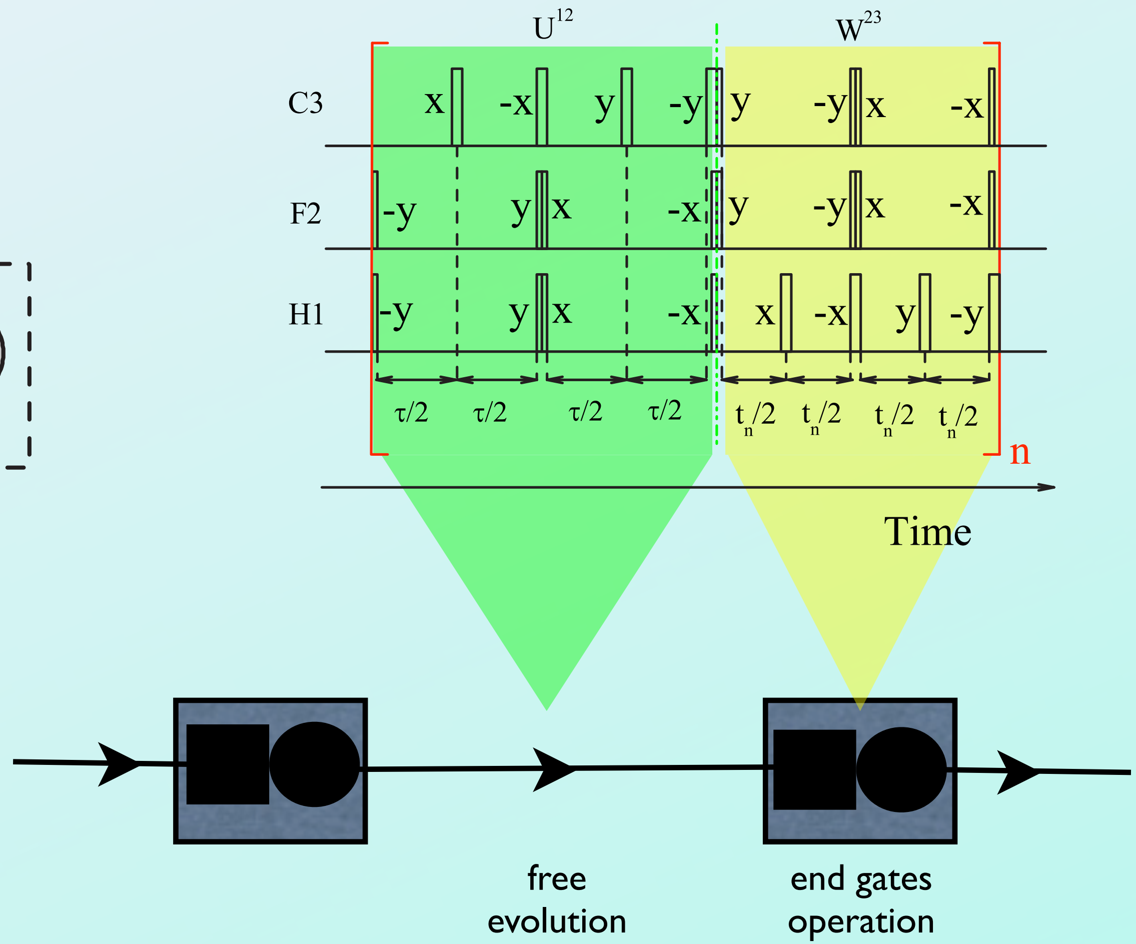


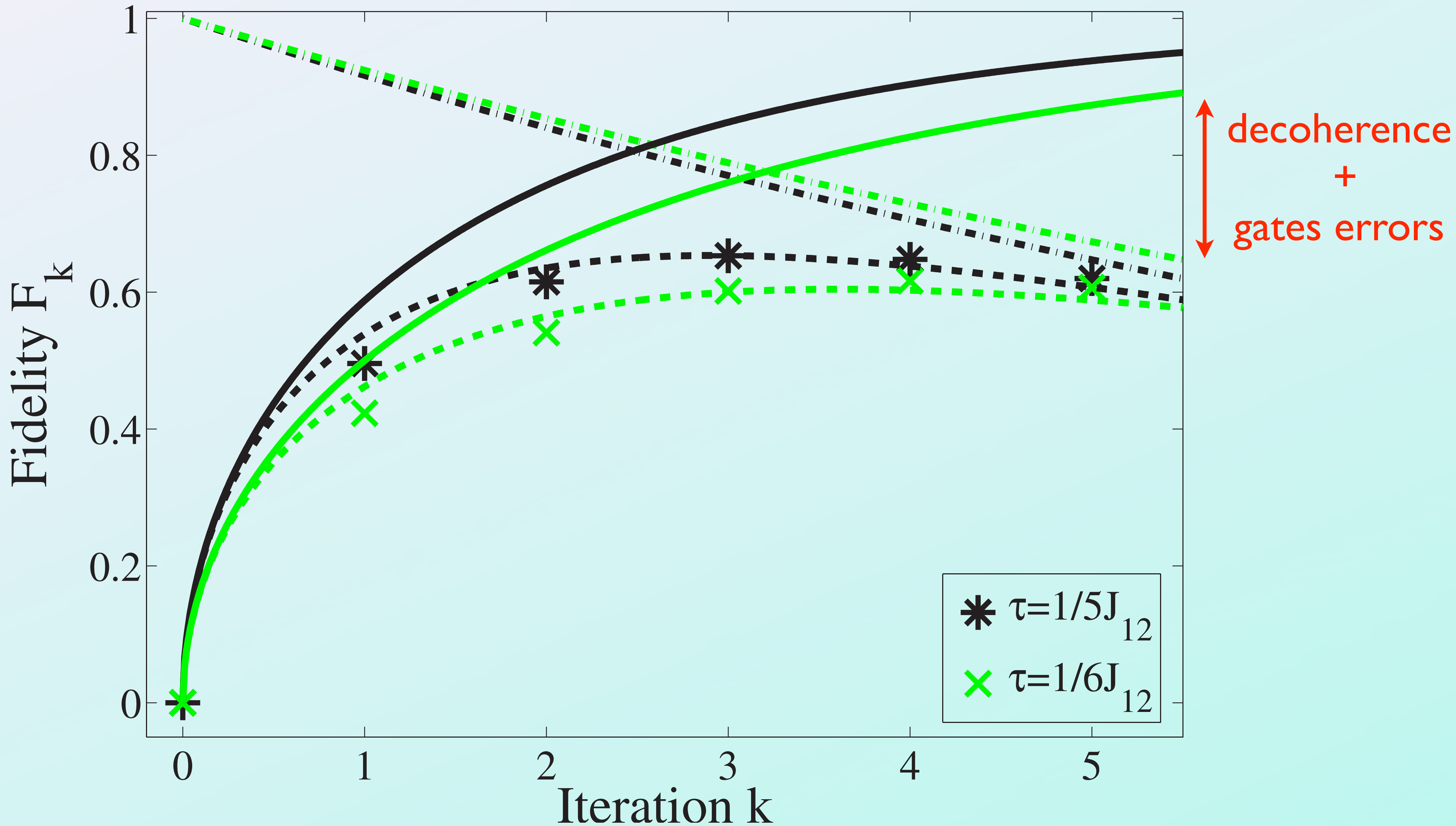
	H1	F2
F2	48.5	
C3	160.8	-195.1

Nucleus	T_1(s)	T_2(s)
H1	3.3	1.1
F2	3.2	1.5
C3	3.7	1.3

$$H_{12} = \frac{1}{2}\pi J_{12}(\sigma_x^1\sigma_x^2 + \sigma_y^1\sigma_y^2)$$






Fidelity F_k
Iteration k
τ=1/5J_{12}
τ=1/6J_{12}
decoherence
+
gates errors

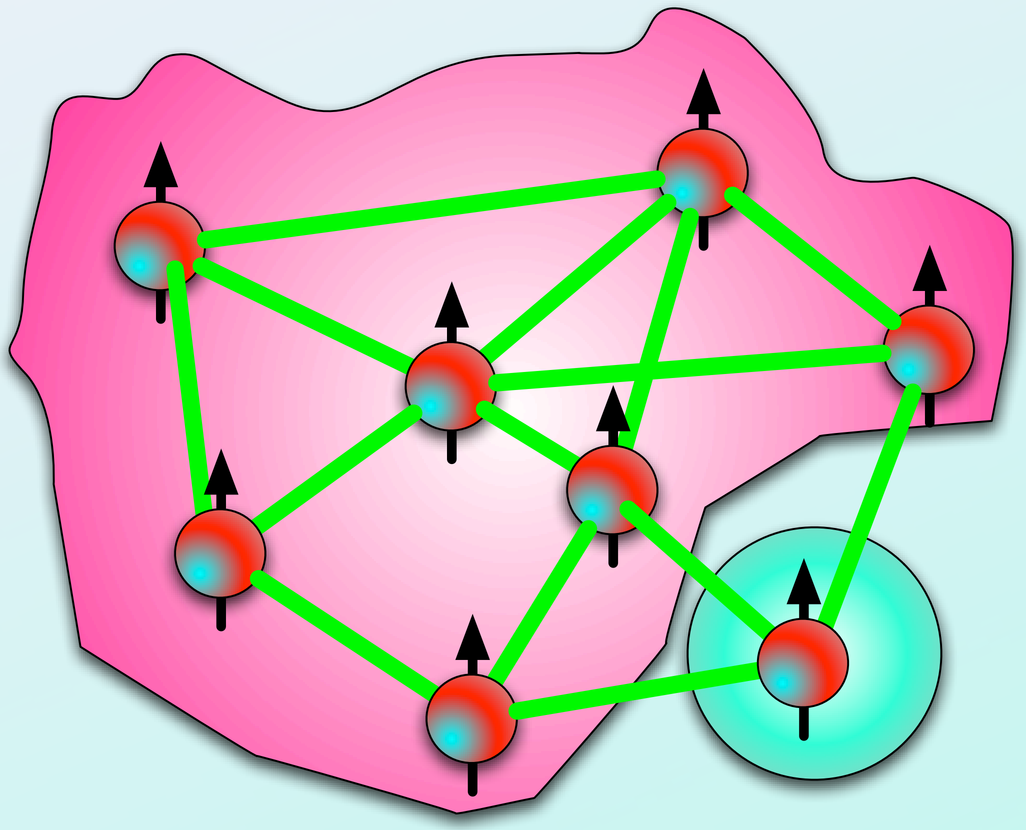
- [x] Quantum control

We know how to extract the information out of the network: what about the reverse procedure? Can we "up-load" the quantum network?

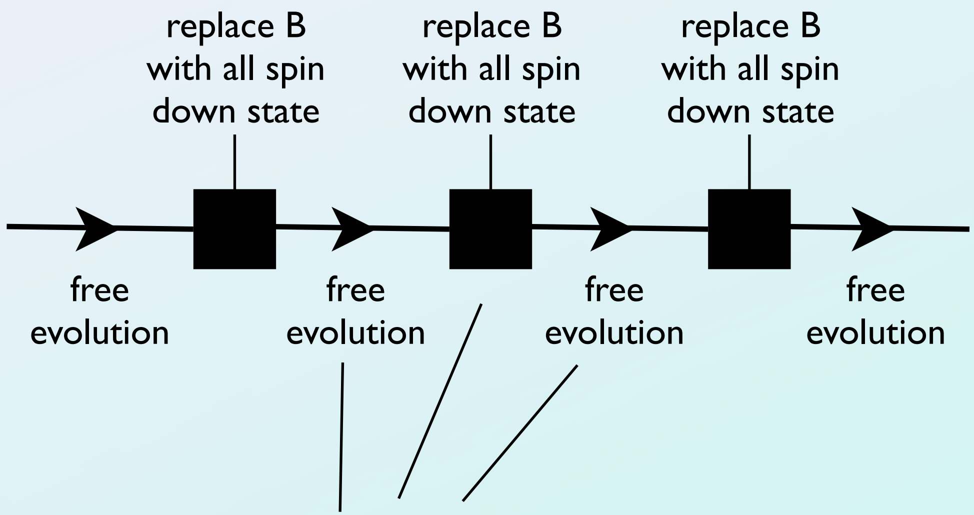
Quantum control

S. Lloyd, Landahl, Slotine PRA69 (2004)

Burgarth, VG PRL (to be published) (2007)



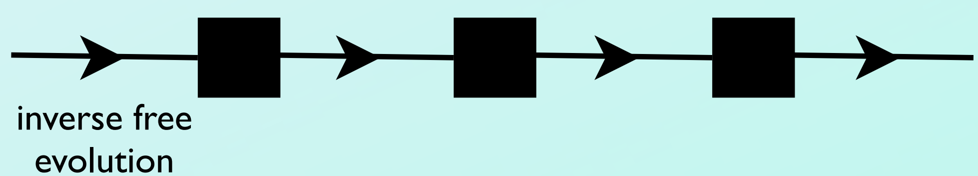
$|\Psi\rangle$
network



$\rho(\Psi)$
memory

$$W \equiv U S_L U S_{L-1} \cdots U S_\ell \cdots U S_1$$

$\rho(\Psi)$
memory



$|\Psi\rangle$
network

$$W^\dagger = S_1 U^\dagger \cdots S_\ell U^\dagger \cdots S_{L-1} U^\dagger S_L U^\dagger$$

- Josephson arrays

Romito, Fazio, Bruder, PRB 71 (2005)
Lyakhov, Bruder, NJP 7 (2005)

- Penning & Pauli traps

Porras, Cirac PRL 92 (2004)
Ciaramicoli, Marzoli, Tombesi, PRA 75 (2007)

- Quantum dots

D'Amico, arXiv: cond-mat/0511470

- NMR

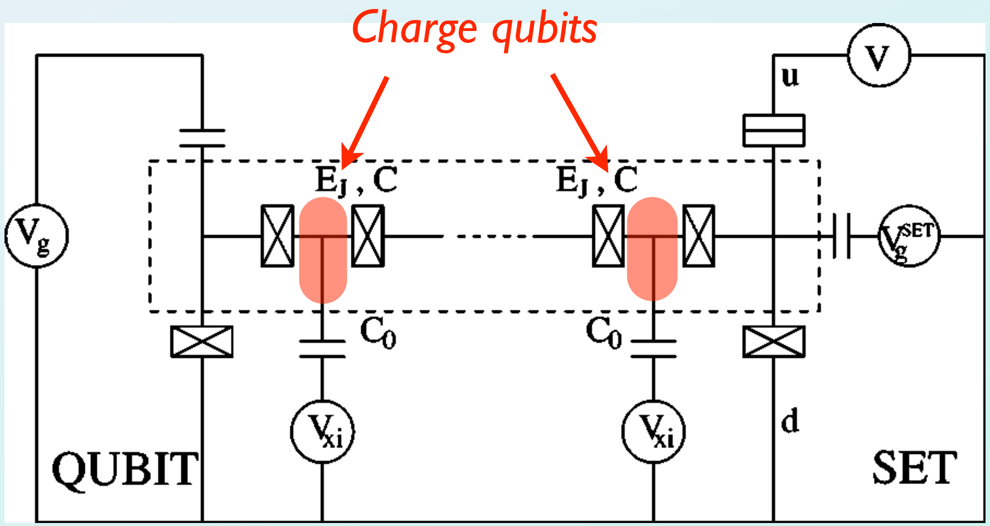
Zhang, Rajendran, Peng, Suter, PRA 76 (2007)
Cappellaro, Ramanathan, Cory arXiv: 0706.0342

- Optical lattices

Duan, , Demler, Lukin, PRL91 (2003)
Jane' et al. IJQC 3 (2006)

Josephson junctions array (I)

Romito, Fazio, Bruder, PRB 71 (2005)



$$|\varnothing\rangle = |0\rangle$$

$$|2e\rangle = |1\rangle$$

$$(E_C \gg E_J)$$

Quantum Phase Model Hamiltonian

inverse capacitance matrix

Josephson tunneling

$$H = \frac{1}{2}\sum_{i,j}(q_i - q_x)\, U_{ij}\, (q_j - q_x) - E_J \sum_{\langle i,j \rangle} \cos\left(\phi_i - \phi_j\right)$$

charging energy

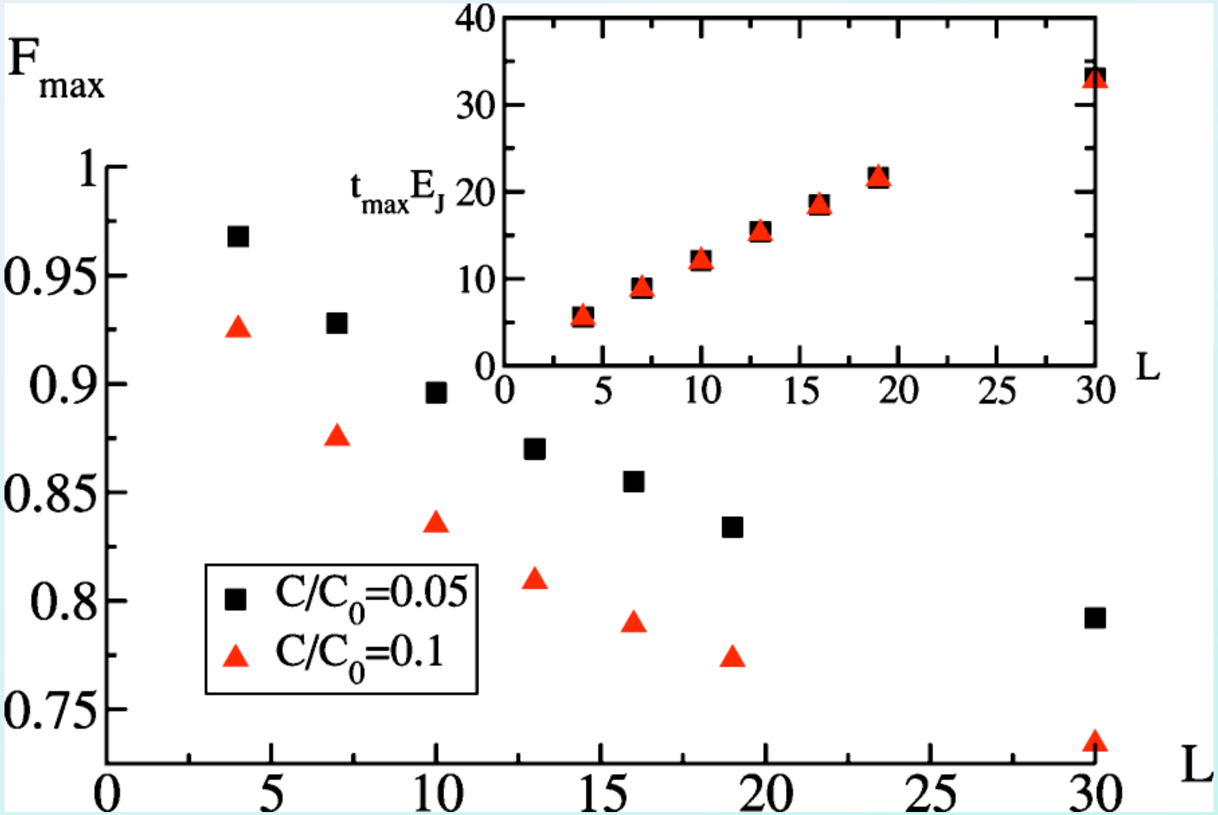
charge regime

$$H = \sum_{ij} U_{ij}\sigma_z^i\sigma_j^z - \frac{E_J}{2}\sum_i(\sigma_x^i\sigma_x^j + \sigma_y^i\sigma_y^j) - \sum_j h_j\sigma_z^j$$

long range interaction

XXZ model (anisotropic Heisenberg model)

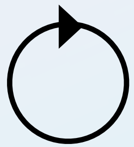
Bruder, Fazio, Schon, PRB47 (1993)

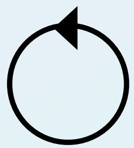


F_{max}
1
0.95
0.9
0.85
0.8
0.75
0
5
10
15
20
25
30
L
C/C_0=0.05
C/C_0=0.1
$t_{max}E_J$
40
30
20
10
0
0
5
10
15
20
25
30
L

Josephson junctions array (II)

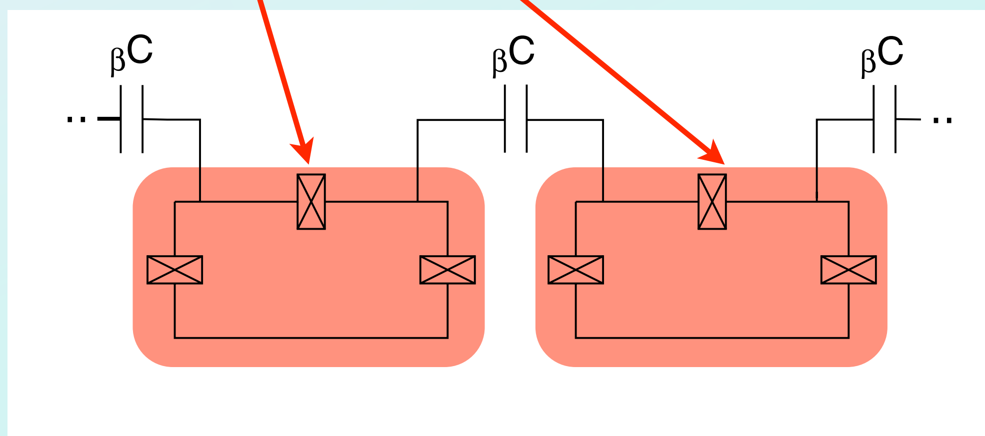
Lyakhov, Bruder, NJP 7 (2005)

 $= |0\rangle$

 $= |1\rangle$

$$(E_c \ll E_J)$$

flux qubits



XXZ model (anisotropic Heisenberg model)

inductive coupling

$$H = -\sum_{i=2}^{N}[J_{xy}(\sigma_i^+\sigma_{i-1}^- + \sigma_i^-\sigma_{i-1}^+) + J_z\sigma_i^z\sigma_{i-1}^z] - \sum_{i=1}^{N}(\Delta\sigma_i^x + B\sigma_i^z)$$

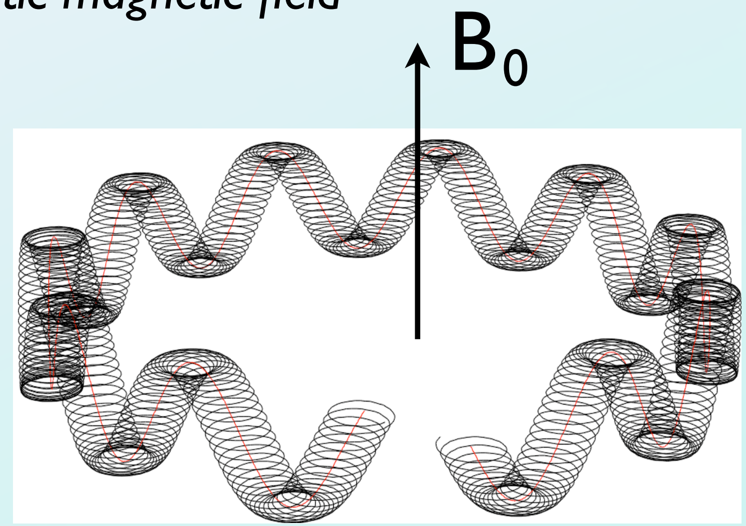
capacitive coupling

tunneling

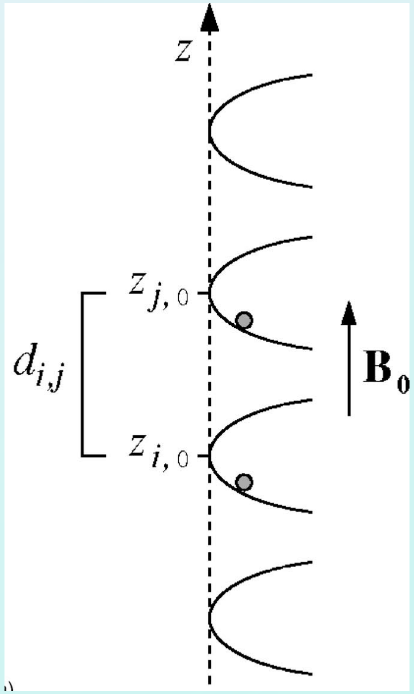
Penning Traps

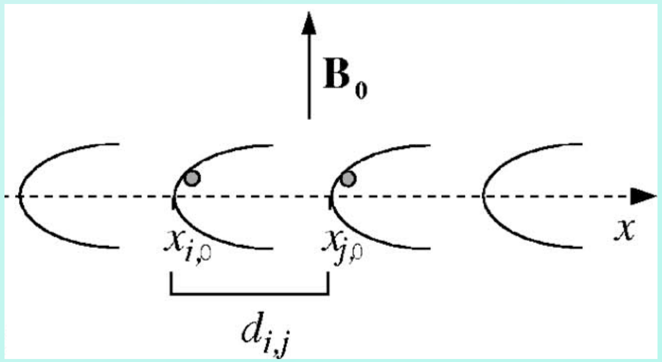
Ciaramicoli, Marzoli, Tombesi, PRA 75 (2007)

electric static potential
(quadrupole potential)
+
static magnetic field

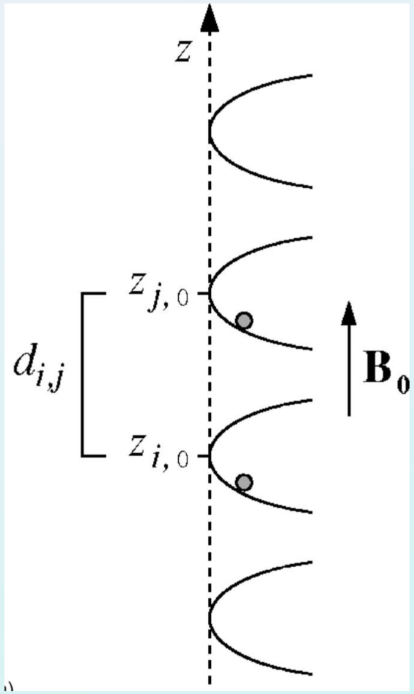


magnetron, cyclotron, axial
oscillations





Castrejon-Pita, Thompson PRA 72 (2005)



Interaction among electrons: coulomb repulsion

"effective" spins-spins coupling

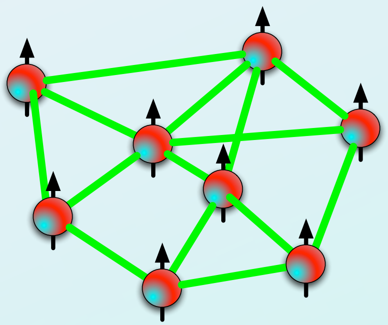
$$H'_s \simeq \sum_{i=1}^{N} \frac{\hbar}{2}\omega_s \sigma_i^z - \hbar \sum_{i>j}^{N} \left(2J_{i,j}^z \sigma_i^z \sigma_j^z - J_{i,j}^{xy}\sigma_i^x \sigma_j^x - J_{i,j}^{xy}\sigma_i^y \sigma_j^y\right),$$

long range interactions

$$J_{ij} \propto d_{ij}^{-3}$$

Zoology of Spin Hamiltonians

XXZ coupling



$$H = -\sum_{\langle i,j\rangle} J_{ij}\left(\sigma_x^i\sigma_x^j + \sigma_y^i\sigma_y^j + \Delta\sigma_z^i\sigma_z^j\right) - \sum_{i=1}^{N} B_i\sigma_z^i$$

exchange coupling: $2\left(\sigma_+^i\sigma_-^j + \sigma_-^i\sigma_+^j\right)$

anisotropy term

local fields

Heisenberg	$\Delta = 1$	$J_{ij} \geqslant 0$ ferro $J_{ij} \leqslant 0$ anti-ferro
XX	$\Delta = 0$	
Ising	NO EXCHANGE	